CS 624: Analysis of Algorithms

Assignment 7

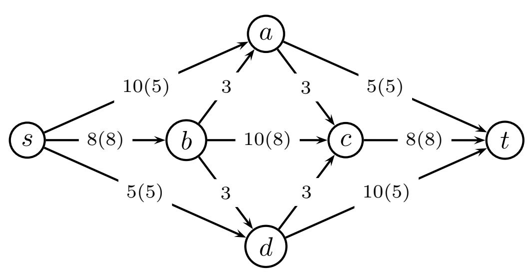
Due: Dec. 13, 2024

1. A directed graph G = (V, E) is said to be semi-connected if, for all pairs of vertices $u, v \in V$ we have $u \rightsquigarrow v$ or $v \rightsquigarrow u$ path (or both).

   Give an efficient algorithm to determine whether or not any directed acyclic graph (DAG) $G = (V, E)$ is semi-connected

2. Professor Bacon wants to rewrite the strongly connected components algorithm and use the original graph (rather than the transpose) in the second DFS run and scan the vertices in *increasing* finish time rather than decreasing. Does this modified algorithm always produce the correct results?

3. **Max-Flow-Min-Cut:** Given the following flow network on which an s-t flow has been computed. The capacity of each edge appears as a label on the edge, and the numbers in parentheses give the amount of flow sent on each edge. (Edges without parentheses – specifically, the four edges of capacity 3 – have no flow being sent on them.)



   (a) What is the value of this flow? Is this a maximum (s,t) flow in this graph?

   (b) Find a minimum s-t cut in the flow network and also say what its capacity is.

4. Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counter example:

   Given an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e ;and let (A, B) be a minimum $s - t$ cut with respect to these capacities $\{c_e | e \in E\}$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum $s - t$ cut with respect to these new capacities $\{1 + c_e | e \in E\}$.

5. Exercise 1.1 (page 3 in the NP notes).

6. Exercise 2.1 (page 9)

7. Exercise 3.6 (page 16)

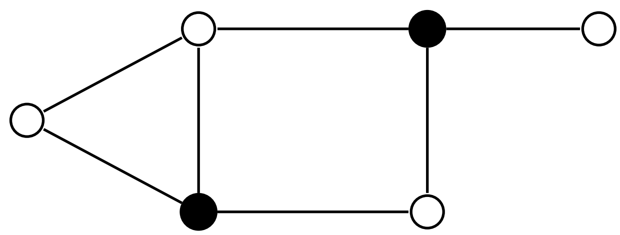
8. Problem name: HITTING SET
Instance: A collection C of subsets of a set S together with a positive integer K.
Question: Does S contain a hitting set for C of size K or less – that is, a subset $S' \subseteq S$ with $|S'| \leq K$ and such that S' contains at least one element of each set $c \in C$?
Prove that HITTING SET is NP-complete.
To do this you need to do two things:

   (a) Prove that HITTING SET is in NP. This should be extremely easy.

   (b) Prove that some problem that is already known to be NP-complete polynomially reduces to HITTING SET.

   Hint: In this case, I suggest you prove that $VC \leq_P$ HITTING SET.

9. A *dominating set* for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. Here is an example (the dominating set is colored in black). Question: Does G contain a dominating set of size K or less?



   (a) Show that the dominating set is in NP

   (b) Show that it is NP complete by constructing a reduction from Vertex Cover (these are indeed similar problems).