THEORY OF COMPUTATION
Problem session - 2

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UMB

1 Problems

2 Solutions

1. Write a program fragment that exchanges the value of two variables. In other words, if $Z_1 = a$ and $Z_2 = b$, after executing this fragment we have $Z_1 = b$ and $Z_2 = a$.
2. Write a function in $\mathcal{S}$ that computes the remainder of the division of m by n.
3. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is even,} \\ 0 & \text{if } x \text{ is odd.} \end{cases}$$

   Write a program in $\mathcal{S}$ that computes f.
4. Let f be a partial function such that $f(x) = 1$ if x is even, and $f(x) \uparrow$ if x is odd. Write a program in $\mathcal{S}$ that computes f.

Problem 1: Write a program fragment that exchanges the value of two variables. In other words, if $Z_1 = a$ and $Z_2 = b$, after executing this fragment we have $Z_1 = b$ and $Z_2 = a$.
The following program fragment $\mathcal{Q}(Z_1, Z_2)$ solves the problem:

$$Z_2 \leftarrow Z_1 + Z_2$$
$$Z_1 \leftarrow Z_2 - Z_1$$
$$Z_2 \leftarrow Z_2 - Z_1$$

An example of the sequence of states of this fragment:

Z_1	Z_2	
5	3	initial state
5	8	after executing $Z_2 \leftarrow Z_1 + Z_2$
3	8	after executing $Z_1 \leftarrow Z_2 - Z_1$
3	5	after executing $Z_2 \leftarrow Z_2 - Z_1$.

Problem 2: Write a function in $\mathcal{S}$ that computes the remainder of the division of m by n.
Examples: $f(20,7) = 6$, $f(7,20) = 7(7 = 20 * 0 + 7)$.

$$
\begin{array}{ll}
[B] & \text{IF } (X_1 \leqslant X_2) \text{ GOTO } A \\
 & X_1 \leftarrow X_1 - X_2 \\
 & \text{GOTO } B \\
[A] & Y \leftarrow X_1 \\
 & \text{GOTO } E
\end{array}
$$

An example of the sequence of states of this fragment:

X_1	X_2	Y
20	7	0initial state
13	7	0
6	7	0
7	7	6

Problem 3: Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is even,} \\ 0 & \text{if } x \text{ is odd.} \end{cases}$$

Write a program in $\mathcal{S}$ that computes f

Solution:

$$
\begin{array}{ll}
 & \text{IF } X = 0 \text{ GOTO } B \\
 & \text{IF } X = 1 \text{ GOTO } E \\
[A] & X \longrightarrow X - 2 \\
 & \text{IF } X > 2 \text{ GOTO } A \\
 & \text{IF } X = 0 \text{ GOTO } B \\
 & Y \leftarrow 0 \\
 & \text{GOTO } E \\
[B] & Y \leftarrow 1
\end{array}
$$

Problem 4: Let f be a partial function such that $f(x) = 1$ if x is even, and $f(x) \uparrow$ if x is odd. Write a program in $\mathcal{S}$ that computes f.

Solution:

$$
\begin{array}{ll}
 & \text{IF } X = 0 \text{ GOTO } B \\
 & \text{IF } X = 1 \text{ GOTO } C \\
[A] & X \longrightarrow X - 2 \\
 & \text{IF } X > 2 \text{ GOTO } A \\
 & \text{IF } X = 0 \text{ GOTO } B \\
[C] & \text{GOTO } C \\
[B] & Y \leftarrow 1
\end{array}
$$