

Homework 3

Posted: October 21, 2024

Due: November 6, 2024

1. Find the program \mathcal{P} such that $\#(\mathcal{P}) = 575$.
2. Let $\text{HALT}^1(x)$ be the predicate defined as $\text{HALT}^1 = \text{HALT}(\ell(x), r(x))$. Show that HALT^1 is not computable.
3. Let $f(x_1, \dots, x_n)$ be computed by program \mathcal{P} where $\#(\mathcal{P}) = p$ and suppose that for some primitive recursive function g ,

$$\text{STP}^{(n)}(x_1, \dots, x_n, p, g(x_1, \dots, x_n))$$

is TRUE for all x_1, \dots, x_n . Show that $f(x_1, \dots, x_n)$ is primitive recursive.

4. Let $B = \{f(n) \mid n \in \mathbb{N}\}$, where f is a strictly increasing function (that is, $f(n+1) > f(n)$ for all n). Prove that B is recursive, that is, P_B is computable.
5. Show that there is no computable function $f(x)$ such that $f(x) = \Phi(x, x) + 1$ whenever $\Phi(x, x)$ is defined.