Homework 2

Posted: February 24, 2025
Due: March 12, 2025

1. Let $A = \{0, 1\}$ be an alphabet that consists of two binary digits. Denote by $f(x)$ the numerical equivalent of x, as we did in class. Design a dfa that accepts the set of words $\{x \in \{0, 1\}^* \mid f(x) \text{ is a multiple of } 6\}$.

2. Construct *deterministic* finite automata that accept the following languages over the alphabet $A = \{a, b, c\}$:

   (a) The set of all words that begin with ab and end with ba. Note that among these words is the word aba.

   (b) The set $\{bab\}$.

   (c) The set $A^* - \{bab\}$.

3. Construct *non-deterministic* finite automata that accept the following languages over the alphabet $A = \{a, b, c\}$:

   (a) The set of all words that begin with ab and end with ba.

   (b) The set $\{bab\}$.

   (c) The set $A^* - \{bab\}$.

4. Prove or disprove the following statements. Proving requires an argument; disproving requires a counterexample.

   (a) Every language is contained in a regular language.

   (b) Every nonempty language contains a nonempty regular language.

   (c) The union of a collection of regular languages is a regular language.

(d) If L_0, L_1 are regular languages and $L_0 \subseteq L \subseteq L_1$, then L is a regular language.

5. Let A be an alphabet and let $a \in A$ be a symbol. If k is a natural number, construct a nondeterministic finite automaton that accepts the language $L_{k,a} = \{uav \mid u, v \in A^* \text{ and } |v| = k\}$.