

Homework 3

Posted: March 24, 2025

Due: April 12, 2025

1. Let $A = \{a, b\}$ be an alphabet. Compute the minimal dfa capable of recognizing the language A^*bA^+a .
2. Prove that the language $\{a^n b^{10n} c^{20n} \mid n \in \mathbb{N}\}$ is not regular.
3. Let $G = (\{S, X, Y, Z\}, \{a, b\}, S, \{S \rightarrow XYZ, X \rightarrow SYZ, Y \rightarrow SXZ, X \rightarrow a, Y \rightarrow b, Z \rightarrow a\})$ be a context-free grammar. Prove that if $x \in L(G)$, the length of x has the form $3 + 2k$, where $k \geq 0$.

Hint: Use induction on the length of the derivation $S \xrightarrow{*}_G x$.

4. Give an example of a non-regular language such that $\text{PREF}(L)$, $\text{SUFF}(L)$, and $\text{INFIX}(L)$ are all regular languages.
5. Consider the context-free grammar $G = (\{S, X, Y\}, \{a, b\}, S, P)$, where the set of productions P is given by

$$P = \{S \rightarrow aXb, S \rightarrow Yb, X \rightarrow YaS, X \rightarrow b, Y \rightarrow bX, Y \rightarrow a\}.$$

- (a) Prove that the word $x = abbaabb$ belongs to $L(G)$ by constructing a derivation d for x . Construct the derivation tree T that corresponds to d .
- (b) Give the leftmost and the rightmost derivations that corresponds to T .