Discrete Mathematics
Exam 1

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1. Consider the word MISSISSIPPI. (It's a favorite for this kind of problem.) How many ways are there to permute the letters if all four Is or all four Ss are together? (IIII, SSSS .)

   I should not have to remind you that in mathematics and in computer science, "or" means "and/or".

2. 18 people have gathered for dinner. Tables at the restaurant seat 6.

   (a) How many ways are there to divide the 18 people into three groups of 6?

   (b) If half the 18 people are women and half men, how many ways are there to divide them so that each table has the same number of women and men?

   (c) How many ways are there to seat six people at a round table if all that matters is who each person's neighbors are?

3. Imagine a test with 20 questions where each question has two possible answers. They are not true/false questions – one or both or neither answer might be correct. Here is an example:

   Is discrete mathematics fun

   - for mathematics students?
   - for computer science students?

   (a) Count the number of possible ways to answer all the questions on that test.

   (b) Which of the following is a good estimate of your count?

   $$10^6 \quad 10^9 \quad 10^{12} \quad 10^{15} \quad 10^{18} \quad \text{none of these}$$

4. A parking lot has n spaces in a row. Cars arrive and fill spaces at random. Then Auntie Em drives up in her SUV, which needs two adjacent spaces.

   (a) If exactly two spaces are free, what is the probability that she is able to park?

   (b) Suppose f spaces are free. (In the previous question $f = 2$.) What is the minimum value of f that guarantees that Auntie Em can park? (Explain how you know you've answered the question correctly.)

   (c) (Optional, hard, take home). Answer the first question if there are f empty spaces. (You've just done $f = 2$, and found values of f that make the probability 1.) Check that your answer agrees with your answer to the first question. Evaluate your expression when $n = 16$ and $f = 4$.