

Discrete Mathematics

Homework 1

Ethan Bolker

September 1, 2014

The Preliminaries on this page are due Thursday, September 4.
The Exercises on the following pages are due Thursday, September 11.

Preliminaries

- Send me email (to `eb at cs dot umb dot edu`) from the email address you want me to use for the class mailing list.
- Complete the questionnaire at <http://www.cs.umb.edu/~eb/320/questionnaire.pdf> by editing and compiling the \LaTeX source at <http://www.cs.umb.edu/~eb/320/questionnaire.tex>. Turn in hard copy of the pdf that includes \LaTeX source – as does the questionnaire.

Answering the questions is easy. I'm asking more than that. To answer them in the form I require you'll have to begin to learn \TeX – in particular, \LaTeX . That typesetting system is the best in the world for mathematics (and lots of other documents too) and well worth the (sometimes substantial) effort it may take you to learn it. If you go on in mathematics or computer science it may well be the most valuable thing you learn in this course.

To use \TeX you need two kinds of software on your computer. The first is a \TeX *distribution*. You can find links to download one at <http://www.cs.umb.edu/~eb/320/tex>, where I have listed some \TeX resources.

The second is an editor. You can write \TeX with any word processor. I use `emacs`. Even Microsoft Word or Notepad would do. But I recommend something like `TexStudio` (<http://texstudio.sourceforge.net/>), which is a complete environment for writing, compiling and previewing \TeX documents. You can find more information at <http://www.cs.umb.edu/~eb/320/tex>.

Once you get started you'll find it fun (I hope). You can ask me \TeX questions, or ask them at <http://tex.stackexchange.com>.

Exercises

Your written mathematics must be

- logically clear – of course. Remember that the point of your argument is not to convince me (I already know) or even to convince yourself (it may be obvious to you) but to convince *me* that you have convinced *yourself* for good reasons.

- free of spelling and grammatical errors.

There are lots of good thinkers who are poor spellers. If you are one, get a good friend who spells well to proofread your work before you turn it in.

I will make some allowance for non native speakers of English (you clearly do better in my language than I could in yours!), but I expect your English to improve as the semester progresses.

- honest. If you are confused at some point, say as clearly as you can what you are confused about. Don't guess – a lucky correct guess without a good reason may convince you that you understand the problem when you don't really. An honest description of what puzzles you will teach you more, and let me teach you more.
- uncluttered. The hard copy pdf you turn in should include the statements of the questions you're answering (so you can read it without referring back to the assignment) but not the instructions like these.

1. How do the writing hints above apply particularly to you? (The point of this question is to force you to *read* those hints, rather than just skip them and start on the mathematics.)

2. Two interesting alphabets.

(a) How many 10 letter words can be written with a 2 letter alphabet?

(b) How many 3 letter words can be written with a 10 letter alphabet?

Note that in the L^AT_EX source the numbers 2, 3 and 10 are written in math mode, surrounded by \$. . . \$. That ensures typographic correctness when fonts change. In particular, they're not in italics here, as 2, 3 and 10 would be.

3. A number written in base 2 has about 3 times as many binary digits (bits) as the number of (decimal) digits in its base 10 representation.

(a) Check this for a few interesting numbers.

(b) Explain why this follows from the answer to the previous question.

(c) Explain why this follows from the value of $\log_2(10)$ or $\log_{10}(2)$. (What is the relation between those two numbers?)

4. Why is the number of $2n$ letter words on a k letter alphabet the square of the number of n letter words? You should be able to answer this two ways, one using a known formula, the other which works even if you have *no idea* what the formula is for n letter words.

5. Consider the word APFELS.¹

- How many ways are there to rearrange these 6 letters?
- How many ways are there to rearrange these 6 letters so that a P comes before an E?

6. Answer the questions in the last Exercise for the word APPLES.

7. How many permutations are there of the numbers $\{0, 1, \dots, 99\}$? In how many of these do the numbers $\{1, 11, 21, \dots, 91\}$ appear in order? Use Stirling's formula to estimate each of these counts.

8. Counting valid phone numbers.

¹That's "apples" in German.

(a) In the 1960s a valid (North American) ten digit telephone number had the form NYX NNX XXXX where

- X is one of the digits 0, 1, . . . 9,
- N is one of the digits 2, 3, . . . 9,
- Y is one of the digits 0, 1.

How many valid telephone numbers were there then?

(b) Now the form is NXX NXX XXXX. How many valid numbers are there now?

(c) If you answered the previous question by a direct count, answer it again by thinking about how much larger it should be than the answer to the first question. If you did it that way first, answer it again with a direct count.

(d) Were there enough numbers then? Are there enough numbers? Will there always be enough numbers? Write a paragraph or two about these questions. Use a round estimate for the population of North America.

9. Watch this space – more to come.

Here is the L^AT_EX source for this document. You can cut it from the pdf and use it to start your answers. I used the `\jobname` macro for the source file name, so you can call your file by any name you like.

```
% Math 320 hw1
%
\documentclass{article}
\pagestyle{empty}
\usepackage[textheight=10in]{geometry}

\usepackage{amsmath}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage{verbatim}

\newcommand{\coursehome}
{http://www.cs.umb.edu/~eb/320}

\title{Discrete Mathematics \\  
Homework 1  
}
\author{Ethan Bolker}
\date{September 1, 2014}

\begin{document}

\maketitle

\noindent
The Preliminaries on this page are due Thursday, September 4.

\noindent
The Exercises on the following pages are due Thursday, September 11.

\section*{Preliminaries}

\begin{itemize}

\item Send me email (to \verb!eb at cs dot umb dot edu!) from the  
email address you want me to use for the class mailing list.

\item Complete the questionnaire at  
\url{\coursehome/questionnaire.pdf} by editing and compiling the  
\LaTeX{} source at \url{\coursehome/questionnaire.tex}. Turn in hard  
copy of the pdf that includes \LaTeX{} source -- as does the questionnaire.

Answering the questions is easy. I'm asking more than that. To  
answer them in the form I require you'll have to begin to learn \TeX{}  
-- in particular, \LaTeX. That typesetting system is the best in the  
world for mathematics (and lots of other documents too) and well worth  
the (sometimes substantial) effort it may take you to learn it. If you  
go on in mathematics or computer science it may well be the most  
valuable thing you learn in this course.

To use \TeX{} you need two kinds of software on your computer. The  
first is a \TeX{} \emph{distribution}. You can find links to download  
one at \url{\coursehome/tex}, where I have listed some \TeX{} resources.

The second is an editor. You can write \TeX{} with any word
```

processor. I use `\verb!emacs!`. Even Microsoft Word or Notepad would do. But I recommend something like TexStudio (`\url{http://texstudio.sourceforge.net/}`), which is a complete environment for writing, compiling and previewing `\TeX{}` documents. You can find more information at `\url{\coursehome/tex}`.

Once you get started you'll find it fun (I hope). You can ask me `\TeX{}` questions, or ask them at `\url{http://tex.stackexchange.com}`.

`\end{itemize}`

`\newpage`

`\section*{Exercises}`

Your written mathematics must be

`\begin{itemize}`

`\item` logically clear -- of course. Remember that the point of your argument is not to convince me (I already know) or even to convince yourself (it may be obvious to you) but to convince `\emph{me}` that you have convinced `\emph{yourself}` for good reasons.

`\item`

free of spelling and grammatical errors.

There are lots of good thinkers who are poor spellers. If you are one, get a good friend who spells well to proofread your work before you turn it in.

I will make some allowance for non native speakers of English (you clearly do better in my language than I could in yours!), but I expect your English to improve as the semester progresses.

`\item`

honest. If you are confused at some point, say as clearly as you can what you are confused about. Don't guess -- a lucky correct guess without a good reason may convince you that you understand the problem when you don't really. An honest description of what puzzles you will teach you more, and let me teach you more.

`\item` uncluttered. The hard copy pdf you turn in should include the statements of the questions you're answering (so you can read it without referring back to the assignment) but not the instructions like these.

`\end{itemize}`

`\begin{enumerate}`

`\item` How do the writing hints above apply particularly to you? (The point of this question is to force you to `\emph{read}` those hints, rather than just skip them and start on the mathematics.)

`\item` Two interesting alphabets.

`\begin{enumerate}`

\item How many 10^k letter words can be written with a 2^k letter alphabet?

\item How many 3^k letter words can be written with a 10^k letter alphabet?

\end{enumerate}

\emph{Note that in the `\LaTeX{}` source the numbers 2^k , 3^k and 10^k are written in math mode, surrounded by `\$ \dots \$`. That ensures typographic correctness when fonts change. In particular, they're not in italics here, as 2, 3 and 10 would be.

}

\item A number written in base 2^k has about 3^k times as many binary digits (bits) as the number of (decimal) digits in its base 10^k representation.

\begin{enumerate}

\item Check this for a few interesting numbers.

\item Explain why this follows from the answer to the previous question.

\item Explain why this follows from the value of $\log_2(10)$ or $\log_{10}(2)$. (What is the relation between those two numbers?)

\end{enumerate}

\item Why is the number of 2^n letter words on a k^n letter alphabet the square of the number of n letter words? You should be able to answer this two ways, one using a known formula, the other which works even if you have `\emph{no idea}` what the formula is for n letter words.

\item Consider the word `\verb!APFELS!`.
\footnote{That's "apples" in German.}

\begin{itemize}

\item How many ways are there to rearrange these 6^k letters?

\item How many ways are there to rearrange these 6^k letters so that a `\verb!P!` comes before an `\verb!E!?`

\end{itemize}

\item Answer the questions in the last Exercise for the word `\verb!APPLES!`.

\item How many permutations are there of the numbers $\{0, 1, \dots, 99\}$? In how many of these do the numbers $\{1, 11, 21, \dots, 91\}$ appear in order? Use Stirling's formula to estimate each of these counts.

% this is just $C(100,10)$, derived a new way ...

\item Counting valid phone numbers.

\begin{enumerate}

\item In the 1960s a valid (North American) ten digit telephone number had the form `\verb!NYX NNX XXXX!` where

```
\begin{itemize}
\item \verb!X! is one of the digits $0, 1, \ldots 9$,
\item \verb!N! is one of the digits $2, 3, \ldots 9$,
\item \verb!Y! is one of the digits $0, 1$.
\end{itemize}
```

How many valid telephone numbers were there then?

```
\item Now the form is \verb!NXX NXX XXXX!.
How many valid numbers are there now?
```

```
\item If you answered the previous question by a direct count, answer
it again by thinking about how much larger it should be than the
answer to the first question. If you did it that way first, answer
it again with a direct count.
```

```
\item Were there enough numbers then? Are there enough numbers? Will
there always be enough numbers? Write a paragraph or two about
these questions. Use a round estimate for the population of North
America.
```

```
\end{enumerate}
```

```
\item Watch this space -- more to come.
```

```
\end{enumerate}
```

```
\newpage
```

Here is the `\LaTeX{}` source for this document. You can cut it from the pdf and use it to start your answers. I used the `\verb!\jobname!` macro for the source file name, so you can call your file by any name you like.

```
\verbatiminput{\jobname}
```

```
\end{document}
```