History of Mathematics
Homework 2

Ethan Bolker

February 7, 2014

This homework is due Thursday, February 13. I may add one question based on the Lobachevsky reading.

1. Prove

   Theorem. *The amount by which the sum of the angles of a spherical triangle exceeds π is proportional to the area of the triangle.*

   Proof. Proof goes here, in a `proof` environment. Note the "QED" symbol □ (sometimes called a *halmos*) that automatically appears at the end.

   In the last page of his autobiography, Paul R. Halmos writes:

   > My most nearly immortal contributions are an abbreviation and a typographical symbol. I invented "iff", for "if and only if" - but I could never believe that I was really its first inventor. I am quite prepared to believe that it existed before me, but I don't know that it did, and my invention (re-invention?) of it is what spread it thorugh the mathematical world. The symbol is definitely not my invention it appeared in popular magazines (not mathematical ones) before I adopted it, but, once again, I seem to have introduced it into mathematics. It is the symbol that sometimes looks like □, and is used to indicate an end, usually the end of a proof. It is most frequently called the "tombstone", but at least one generous author referred to it as the "halmos".

   I found this at `http://jeff560.tripod.com/mathsym.html`.

   □

   You can figure out your own proof, or find one somewhere and rewrite it – as long as you make it clear that you understand what you've written. Don't just copy/paste. If you use work from elsewhere, tell me where.

   You need not use historical methods. Modern tools are just fine.

2. Research the history of the theorem you just proved. Did the Greeks know it? Does it appear in Euclid's *Elements*?

3. Find out about the term *pons asinorum*. You can start your google search at Wikipedia, but don't just cut and paste from that page. I've read it and don't need your reprise. Tell me something new and interesting.

4. Prove

   Theorem. *If the sum of the angles of every right triangle is two right angles then the sum of the angles of every triangle is two right angles.*

   Hint (which you shouldn't need): you can cut an arbitrary triangle into two right triangles.

5. Prove

   Theorem. *If the sum of the angles of every triangle is two right angles then Euclid's parallel postulate is true.*

   If you can invent a proof of your own, do. But it's OK with me if you look one up somewhere, understand it and present it. There's a link on the course Euclid page that may help.

   Proving that this hypothesis implies HRA in Sacchieri's quadrilateral would suffice.

Here is the LaTeX source for this document. You can cut it from the pdf and use it to start your answers. I used the `\jobname` macro for the source file name, so you can call your file by any name you like.

```
% Math 370 hw2 Spring 2014
%
\documentclass{article}
\pagestyle{empty}

\usepackage{amsmath}
\usepackage{amsthm}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage{verbatim}

%% create an environment for theorems
\newtheorem*{thm}{Theorem}

\newcommand{\coursehome}
{http://www.cs.umb.edu/~eb/370}

\title{History of Mathematics \\
Homework 2
}
\author{Ethan Bolker}

\begin{document}

\maketitle

This homework is due Thursday, February 13. I may add one question
based on the Lobachevsky reading.

\begin{enumerate}

\item Prove
\begin{thm}
The amount by which the sum of the angles of a spherical triangle
exceeds $\pi$ is proportional to the area of the triangle.
\end{thm}
\begin{proof}
Proof goes here, in a \verb!proof! environment. Note the ``QED''
symbol \qedsymbol{} (sometimes called a \emph{halmos})
that automatically appears at the end.

In the last page of his autobiography, Paul R. Halmos writes:
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\begin{quotation}
    My most nearly immortal contributions are an abbreviation and a
    typographical symbol. I invented ‘‘iff’’, for ‘‘if and only if’’
    -\ but I could never believe that I was really its first
    inventor. I am quite prepared to believe that it existed before
    me, but I don’t know that it did, and my invention
    (re-invention?) of it is what spread it thorugh the mathematical
    world. The symbol is definitely not my invention \- it appeared in
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\end{quotation}

I found this at \url{http://jeff560.tripod.com/mathsym.html}.

\end{proof}

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it -- as long as you make it clear that you understand what you’ve
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\item Find out about the term \emph{pons asinorum}. You can start your
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angles then the sum of the angles of every triangle is two right angles.
\end{thm}

Hint (which you shouldn’t need): you can cut an arbitrary triangle
into two right triangles.

\item Prove
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\begin{thm}
If the sum of the angles of every triangle is two right angles then
Euclid's parallel postulate is true.
\end{thm}

If you can invent a proof of your own, do. But it's OK with me if you
look one up somewhere, understand it and present it. There's a link on
the course Euclid page that may help.

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\newpage

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\verbatiminput{\jobname}

\end{document}
```