HW1 Solution: Logic and Proofs

Assigned: 13 July 2020 **Due:** 19 July 2020

1. **Exercise 16 (parts d,e,f) page 14**

   d) $p \wedge \neg q \wedge r$ e) $(p \wedge q) \rightarrow r$ f) $r \leftrightarrow (q \vee p)$

2. **Exercise 36 (parts a,b,c) page 16**

   a,b,c)

p	q	$\neg p$	$\neg q$	$p \oplus p$	$p \oplus \neg p$	$p \oplus \neg q$
1	1	0	0	0	1	1
1	0	0	1	0	1	0
0	1	1	0	0	1	0
0	0	1	1	0	1	1

3. **Exercise 18 page 38**

   It's not a tautology. We can show it by truth table or by using equivalences:

$$\big(\neg p \wedge (p \rightarrow q)\big) \rightarrow \neg q \equiv \neg\big(\neg p \wedge (p \rightarrow q)\big) \vee \neg q \equiv p \vee \neg(p \rightarrow q) \vee \neg q$$
$$\equiv p \vee (p \wedge \neg q) \vee \neg q \equiv p \vee \neg q$$

   So if we choose $p = 0$ and $q = 1$, then $p \vee \neg q \equiv F$

4. **Exercise 24 page 38**

$$\neg(p \oplus q) \equiv \neg((p \wedge \neg q) \vee (\neg p \wedge q)) \equiv \neg(p \wedge \neg q) \wedge \neg(\neg p \wedge q)$$
$$\equiv (\neg p \vee q) \wedge (p \vee \neg q) \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$$

5. **Exercise 36 (parts a,c) page 58**

   a) $\neg\forall x(-2 < x < 3) \equiv \exists x\big((x \leq -2) \vee (x \geq 3)\big)$

   c) $\neg\exists x(-4 \leq x \leq 1) \equiv \forall x((x < -4) \vee (x > 1))$

6. **Exercise 52 page 59**

   Suppose P(x) is "x is odd" and Q(x) is "x is even". We have:

   $\forall x P(x) \vee \forall x Q(x) \equiv F \vee F \equiv F$ $\forall x\big(P(x) \vee Q(x)\big) \equiv T$

So they are not logically equivalent.

7. **Exercise 24 page 71**
   a) There is a real number x (x=0) such that if you add it to any real number y, the result will be equal to y.
   b) Any not negative real number is greater than any negative number.
   c) There are at least 2 not positive real numbers which one of them is bigger than the other one.
   d) For any two real numbers x and y, none of them are zero if and only if their multiplication is not zero.
8. **Exercise 36 page 71**
   a) Suppose L(x,y) is "x lost y thousand dollars in playing lottery".
   The statement: $\forall x \forall y(\neg L(x,y) \vee y \leq 1)$. The negation: $\neg \forall x \forall y(\neg L(x,y) \vee y \leq 1) \equiv \exists x \exists y(L(x,y) \wedge y > 1)$. There is at least one person who lost more than 1 thousand dollars in playing lottery.
   b) Suppose H(x,y) is "x and y chatted".
   The statement: $\exists x \exists y(H(x,y) \wedge \forall z(H(x,z) \rightarrow (y=z))$. The negation: $\forall x \forall y(\neg H(x,y) \vee \exists z(H(x,z) \wedge (y \neq z))$. For any two students x and y in this class, they didn't chat with each other or there is at least one other student who x chatted with him/her as well.
   c) Suppose E(x,y) is "x sent an e-mail to y".
   The statement: $\forall x \forall y \forall z((y=z) \vee \neg E(x,y) \vee \neg E(x,z) \vee \exists s(E(x,s) \wedge (s \neq y) \wedge (s \neq z)))$. The negation: $\exists x \exists y \exists z((y \neq z) \wedge E(x,y) \wedge E(x,z) \wedge \forall s(\neg E(x,s) \vee (s=y) \vee (s=z)))$. There is at least 3 students x, y, and z in this class that x sent an e-mail to y and an e-mail to z and x didn't send any other emails to other students.
   d) Suppose S(x,y) is "x solved exercise y in this book".

The statement: $\exists x \forall y(S(x,y))$. The negation: $\forall x \exists y(\neg S(x,y))$. For any student x in this class, there is at least one question y in this book such that x didn't solve exercise y in this book.

e) Suppose S(x,y) is "x solved exercise y in this book" and C(y,z) is "question y is in section z".

The statement: $\forall x \exists z \forall y(\neg S(x,z) \vee \neg C(y,z))$. The negation: $\exists x \forall z \exists y(S(x,z) \wedge C(y,z))$. There is at least one student who solved at least one question from any section of the book.

9. **Exercise 12 page 83**

By using exercise 11 we should show that the argument form with premises $P_1: (p \wedge t) \rightarrow (r \vee s)$, $P_2: q \rightarrow (u \wedge t)$, $P_3: u \rightarrow p$, $P_4: \neg s$, $P_5: q$ and conclusion r is valid.

Using modus ponens and P_5 and P_2, $(u \wedge t)$ follows. Using simplification and $(u \wedge t)$, follows both u and t. Using modus ponens and P_3 and u, p follows. Using conjunction and p and t, $(p \wedge t)$ follows. Using modus ponens and $(p \wedge t)$ and P_1, $(r \vee s)$ follows. Using disjunctive syllogism and $(r \vee s)$ and P_4, r which is conclusion follows. So we showed that argument form with premises P_1, P_2, P_3, P_4, P_5 and conclusion r is valid. By the result of question 11, we know that the argument form with premises P_1, P_2, P_3, P_4 and conclusion $q \rightarrow r$ is valid.

10. **Exercise 16 page 83**

a) Correct. Using universal instantiation and $\forall x U(x) \rightarrow D(x)$, $U(Mia) \rightarrow D(Mia)$ follows. Using modus tollens and $U(Mia) \rightarrow D(Mia)$ and $\neg D(Mia)$, follows $\neg U(Mia)$

b) Not correct. Because from $\forall x C(x) \rightarrow F(x)$ and $\neg C(Isaac)$, we cannot conclude $\neg F(Isaac)$

c) Not correct. Because from $\forall x A(x) \rightarrow L(Quincy, x)$ and $L(Quincy, 8\ men\ out)$, we cannot conclude $A(8\ men\ out)$.

d) Correct. Using universal instantiation and $\forall x L(x) \rightarrow D(x)$, $L(Hamilton) \rightarrow D(Hamilton)$ follows. Using modus ponens and $L(Hamilton) \rightarrow D(Hamilton)$ and $L(Hamilton)$, $D(Hamilton)$ follows.

11. Exercise 40 page 96

Number 7 is a counterexample. Square of 3 is 9. So we should choose 3 numbers from 0,1, and 2. And it's not possible to write 7 as the sum of the squares of three numbers from 0,1, and 2.

12. Exercise 44 page 96

$(i) \rightarrow (ii)$: By contradiction suppose $1 - n$ is odd. So we have

$1 - n = 2k + 1 \Rightarrow n = -2k \Rightarrow n^2 = 4k^2 = 2k'$. But we know that n^2 is odd. So our first assumption was wrong. Therefore $1 - n$ is even.

$(ii) \rightarrow (iii)$: $1 - n = 2j \Rightarrow n = 1 - 2j \Rightarrow n^3 = -8j^3 + 12j^2 - 6j + 1 = 2h + 1$

$(iii) \rightarrow (iv)$: By contradiction suppose $n^2 + 1$ is odd. So we have

$n^2 + 1 = 2k + 1 \Rightarrow n^2 = 2k \Rightarrow n = 2k' \Rightarrow n^3 = 8k' = 2g$. But we know that n^3 is odd. So our first assumption was wrong. Therefore $n^2 + 1$ is even.

$(iv) \rightarrow (i)$: $n^2 + 1 = 2k \Rightarrow n^2 = 2k - 1 = 2k' + 1$

13. Exercise 10 page 113

$3 = 1 + 2$. It's a constructive proof because we found a witness such that the proposition is true for.