**HW2: Set, Functions, Sequences, Sums, matrices, and Algorithms**

**Assigned:** 19 July 2020 **Due:** 26 July 2020

1. **Exercise 22 page 132**

a) 0 b) 1 c) 2 d) 3

2. **Exercise 28 page 132**

Suppose $(a, b) \in A \times B \longrightarrow (a \in A) \wedge (b \in B) \longrightarrow (a \in C) \wedge (b \in D) \longrightarrow (a, b) \in C \times D \longrightarrow A \times B \subseteq C \times D$

3. **Exercise 12 page 144**

Suppose $x \in A \cup (A \cap B) \longrightarrow x \in A \vee x \in (A \cap B) \longrightarrow (x \in A) \vee (x \in A \wedge x \in B) \longrightarrow (x \in A \vee x \in A) \wedge (x \in A \vee x \in B) \longrightarrow (x \in A) \wedge (x \in A \vee x \in B) \longrightarrow x \in A \longrightarrow A \cup (A \cap B) \subseteq A$

For the other side, suppose $x \in A \longrightarrow x \in A \cup (A \cap B)$

So we have $A \cup (A \cap B) = A$

4. **Exercise 56 page 145**

a) $\bigcup_{i=1}^{\infty} A_i = Z^+$ $\bigcap_{i=1}^{\infty} A_i = \emptyset$

b) $\bigcup_{i=1}^{\infty} A_i = N$ $\bigcap_{i=1}^{\infty} A_i = \{0\}$

c) $\bigcup_{i=1}^{\infty} A_i = R^+$ $\bigcap_{i=1}^{\infty} A_i = (0,1)$

d) $\bigcup_{i=1}^{\infty} A_i = (1, \infty)$ $\bigcap_{i=1}^{\infty} A_i = \emptyset$

5. **Exercise 12 page 162**

a) One-to-on because $f(x) = f(y) \longrightarrow x - 1 = y - 1 \longrightarrow x = y$

b) Not one-to-one because $f(x) = f(y) \longrightarrow x^2 + 1 = y^2 + 1 \longrightarrow x = \pm y$. For example $x = 1$ and $y = -1$. So $x \neq y$

c) One-to-on because $f(x) = f(y) \longrightarrow x^3 = y^3 \longrightarrow x = y$

d) Not one-to-one because $f(x) = f(y) \longrightarrow \left\lceil \frac{x}{2} \right\rceil = \left\lceil \frac{y}{2} \right\rceil$. For example $x = 1$ and $y = 2$. So $x \neq y$

**6. Exercise 22 page 162**

a) It is a bijection because it's both one-to-one and onto.

$f(x) = f(y) \to -3x + 4 = -3y + 4 \to x = y$

$\forall y \exists x(y = -3x + 4 \to x = \frac{4-y}{3})$

b) Not a bijection. Because it's not one-to-one.

$f(x) = f(y) \to -3x^2 + 7 = -3y^2 + 7 \to x^2 = y^2 \to x = \pm y$

c) Not a function. Because you cannot assign -2 to x.

d) It is a bijection because it's both one-to-one and onto.

$f(x) = f(y) \to x^5 + 1 = y^5 + 1 \to x = y$

$\forall y \exists x(y = x^5 + 1 \to x = \sqrt[5]{y-1}$

**7. Exercise 26 (a,b,c) page 178**

a) $a_n = n^2 + 2$. The next three terms are 123, 146, 171

b) $a_n = 7 + 4(n-1)$. The next three terms are 47, 51, 55

c) $a_n =$ binary expansion of $n$. The next three terms are 1100, 1101, 1110

**8. Exercise 34 page 179**

a) $\sum_{i=1}^{3}\sum_{j=1}^{2}(i-j) = \sum_{i=1}^{3}\left(\sum_{j=1}^{2} i - \sum_{j=1}^{2} j\right) = \sum_{i=1}^{3}(2i-3) = 2\sum_{i=1}^{3} i - 9 = 3$

b) $\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j) = \sum_{i=0}^{3}\left(\sum_{j=0}^{2} 3i + \sum_{j=0}^{2} 2j\right) = \sum_{i=0}^{3}\left(3\sum_{j=0}^{2} i + 2\sum_{j=0}^{2} j\right) =$

$\sum_{i=0}^{3}(9i+6) = 9\sum_{i=0}^{3} i + \sum_{i=0}^{3} 6 = 54 + 24 = 78$

c) $\sum_{i=1}^{3}\sum_{j=0}^{2} j = \sum_{i=1}^{3} 3 = 9$

d) $\sum_{i=0}^{2}\sum_{j=0}^{3}(i^2 j^3) = \sum_{i=0}^{2}\left(i^2 \sum_{j=0}^{3} j^3\right) = \sum_{i=0}^{2}\left(i^2 \frac{(3^2 * 4^2)}{4}\right) = 36\sum_{i=0}^{2} i^2 = 180$

**9. Exercise 6 page 194**

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. So we have:

$a + 3d + 2g = 7,\ \ b + 3e + 2h = 1,\ \ c + 3f + 2i = 3,\ \ 2a + d + g = 1,\ \ 2b + e + h = 0,\ 2c + f + i = 3,\ 4a + 3g = -1,\ 4b + 3h = -3,\ 4c + 3i = 7$

Therefore $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

**10. Exercise 20 page 194**

a) $A^{-1} = \begin{bmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$

b) $A^2 = \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix},\ A^3 = \begin{bmatrix} 1 & 18 \\ 9 & 37 \end{bmatrix}$

c) $(A^{-1})^2 = \begin{bmatrix} \frac{11}{25} & -\frac{4}{25} \\ -\frac{2}{25} & \frac{3}{25} \end{bmatrix},\ (A^{-1})^3 = \begin{bmatrix} -\frac{37}{125} & \frac{18}{125} \\ \frac{9}{125} & -\frac{1}{125} \end{bmatrix}$

d) $(A^{-1})^3 = (A^3)^{-1} = \begin{bmatrix} -\frac{37}{125} & \frac{18}{125} \\ \frac{9}{125} & -\frac{1}{125} \end{bmatrix}$

**11. Exercise 2 page 213**

a) It has Definiteness and Effectiveness

b) It has Definiteness and Finiteness

c) It has Definiteness, Correctness, Finiteness, and Effectiveness

d) It has Correctness, Finiteness, and Effectiveness

12. **Exercise 8 page 214**

**procedure** max_even($a_1, a_2, \dots, a_n$: integers)

meven := 0

idx := 0

**for** i := 1 to n

**begin**

**if** $a_i$ is even and $a_i > meven$ **then** $meven \coloneqq a_i$ and idx := i

**end**

**return** idx

13. **Exercise 26 page 229**

a) $O\big(f(x)\big) = O(\max(n^3 . \log n,\ \log n . n^3)) = O(n^3 \log n)$

b) $O\big(f(x)\big) = O(2^n . 3^n) = O(6^n)$

c) $O\big(f(x)\big) = O(\mathrm{n}^{\mathrm{n}} . n!)$

14. **Exercise 40 page 229**

We will show that $\log_b x = O(\log_a x)$ and $\log_a x = O(\log_b x)$.

If you choose $K = 0$ and $C = \log_b a$, then we have $\log_b x \leq C \log_a x = \log_a x\, C = \log_a x \log_b a = \log_b a^{\log_a x} = \log_b x$. So $\log_b x = O(\log_a x)$

If you choose $K = 0$ and $C = \log_a b$, then we have $\log_a x \leq C \log_b x = \log_b x\, C = \log_b x \log_a b = \log_a b^{\log_b x} = \log_a x$. So $\log_a x = O(\log_b x)$

Therefore $O(\log_a x) = O(\log_b x)$

15. **Exercise 2 page 241**

There are two for loop from 1 to n. So it's $O(n^2)$