CS220/MATH320 - Applied Discrete Mathematics

Practice Midterm Exam

Question 1: What do we know?

Let p, q, and r be propositions; p is known to be true, q is known to be false, and r's truth value is unknown at this time. Tell whether each of the following compound propositions is true, is false, or has an unknown truth value at this time by circling the appropriate word.

a) $p \vee r$	true	false	unknown
b) $p \wedge r$	true	false	unknown
c) $p \rightarrow r$	true	false	unknown
d) $q \rightarrow r$	true	false	unknown
e) $r \rightarrow p$	true	false	unknown
f) $r \rightarrow q$	true	false	unknown
g) $(p \wedge r) \leftrightarrow r$	true	false	unknown
h) $(p \vee r) \leftrightarrow r$	true	false	unknown
i) $(q \wedge r) \leftrightarrow r$	true	false	unknown
j) $(q \vee r) \leftrightarrow r$	true	false	unknown

Question 2: Some Simple Tasks…

Show your calculations for each task.

a) Use the prime factorization method to determine gcd(5000, 600).

b) Use the prime factorization method to determine lcm(30, 42).

c) Convert the integer $(1101001)_2$ from binary expansion to decimal expansion.

d) Convert the integer 5322 from decimal expansion to hexadecimal expansion.

Question 3: Rules of Inference

Use rules of inference to show that the arguments below are valid, i.e., that their conclusion follows from their hypotheses. First extract and name all relevant propositions, and then write down all hypotheses and the conclusion in propositional logic notation. Finally, apply the step-by-step method we used in class and list all those steps in your answer.

a) Hypotheses: If there is gas in the car, then I will go to the store. If I go to the store, then I will get a soda. There is gas in the car. Conclusion: I will get a soda.

b) Hypotheses: When Prof. P. gets angry, he fails his entire class. When the entire class fails, the Chancellor gets complaints. When the Chancellor gets complaints, she will either fire Prof. P., cut his salary, or do both. Prof. P. got angry, and he was not fired. Conclusion: Prof. P.'s salary was cut.

Question 4: Complexity

Take a look at the following algorithm written in pseudocode:

```
procedure mystery(a₁, a₂, …, aₙ: integer)
i := 1
while (i < n and aᵢ ≤ aᵢ₊₁)
  i := i + 1
if i = n then print "Yes!"
else print "No!"
```

a) What property of the input sequence $\{a_n\}$ does this algorithm test?

b) What is the computational complexity of this algorithm, i.e., the number of comparisons being computed as a function of the input size n?

c) Provide a reasonable upper bound for the growth of the complexity function by using the big-O notation (no proof necessary).

Question 5 (Bonus): Functions

Explain in your own words (no numbers or equations are necessary) why only bijective functions have an inverse function.