Introduction to Programming in Python

Building a Computer: Logic Circuits

Outline

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The three basic boolean functions: $\texttt{not}(x) = \bar{x}$, $\texttt{or}(x, y) = x + y$, and $\texttt{and}(x, y) = x \cdot y$

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Truth tables for `not`, `or`, and `and` functions

x	$\bar{x}$
0	1
1	0

x	y	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

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The proposition is described by the `implication` function $(x \implies y)$

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0	1	1	
1	0	0	
1	1	1	

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0	0	1	$\bar{x} \cdot \bar{y}$
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0	1	1	$\bar{x} \cdot y$
1	0	0	
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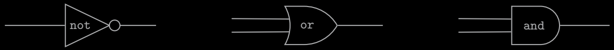
Therefore, $\texttt{implication}(x, y) = \bar{x} \cdot \bar{y} + \bar{x} \cdot y + x \cdot y$

Logic Circuits

Logic Circuits

The circuits (called gates) that implement the `not`, `or`, and `and` functions



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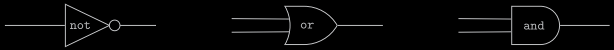
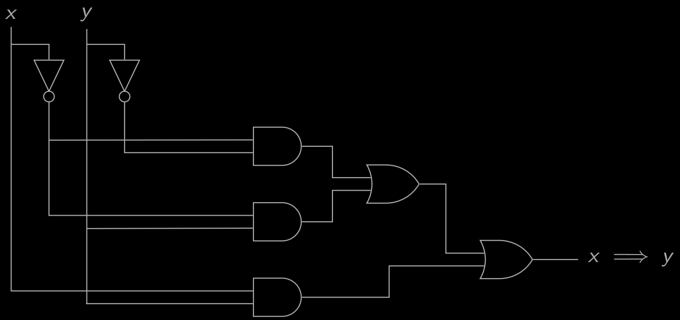
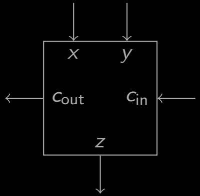


The circuit for the `implication` function $\bar{x} \cdot \bar{y} + \bar{x} \cdot y + x \cdot y$



A full adder (FA) circuit can add two 1-bit numbers (with carry) to produce a 2-bit result



x	y	c_{in}	z	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

An n-bit ripple-carry adder for adding two n-bit numbers is n FA circuits chained together

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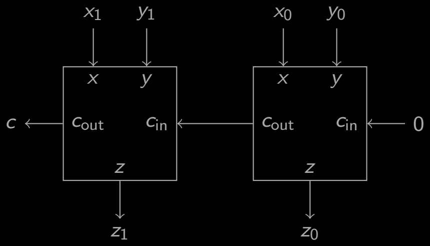
Example (2-bit ripple-carry adder for adding two 2-bit numbers)



Truth table for a `nor` gate (`or` followed by `not`)

x	y	$\overline{x + y}$
0	0	1
0	1	0
1	0	0
1	1	0

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x	y	$\overline{x + y}$
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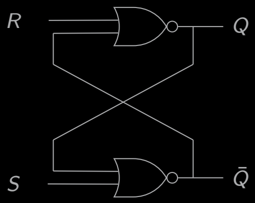
A 1-bit memory circuit, called a latch, built using two `nor` gates



S	R	Q	$\bar{Q}$
0	0	0	1
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1

Truth table for a nor gate (or followed by not)

x	y	$\overline{x+y}$
0	0	1
0	1	0
1	0	0
1	1	0

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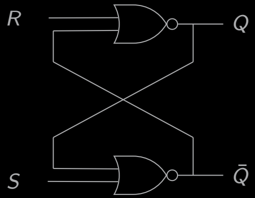

S	R	Q	$\bar{Q}$
0	0	0	1
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1

Billion latches can be combined to produce a 1GB Random Access Memory (RAM) module