Introduction to Compiler Construction

Scanning: Generating a Scanner

Outline

Regular Expressions

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Grouping Rule: both r and (r) describe the same language, ie, $L(r) = L((r))$

Regular Expressions

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Example: given an alphabet $\Sigma = \{a, b\}$

- $aa|ab|ba|bb$ is the language of all two-symbol strings over the alphabet
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- $(a|b)*ab$ is the language of all strings of a's and b's ending in ab

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Example (integer literals in *j*--): `( "0"..."9" ) ( "0"..."9" )*`

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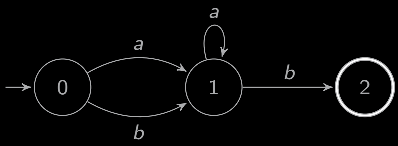
An FSA is a quintuple $A = (\Sigma, S, s_0, F, M)$, where

1. Σ is the input alphabet
2. S is a set of states
3. $s_0 \in S$ is a special start state
4. $F \subseteq S$ is a set of final states
5. M is a set of moves (aka transitions) of the form $m(r, a) = s$, where $r, s \in S$ and $a \in \Sigma$

Finite State Automata

Finite State Automata

Example (an FSA A that recognizes $L((a|b)a*b)$)



Formally, $A = (\Sigma, S, s_0, F, M)$, where $\Sigma = \{a, b\}$, $S = \{0, 1, 2\}$, $s_0 = 0$, $F = \{2\}$, and M is

r	a	$m(r, a)$
0	a	1
0	b	1
1	a	1
1	b	2

Non-deterministic Versus Deterministic Finite State Automata

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A non-deterministic finite state automaton (NFA) is one that allows

- An ϵ-move defined on the empty string ϵ, ie, $m(r, \epsilon) = s$
- More than one move from a state r on an input symbol a, ie, $m(r, a) = s$ and $m(r, a) = t$, where $s \neq t$

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- More than one move from a state r on an input symbol a, ie, $m(r, a) = s$ and $m(r, a) = t$, where $s \neq t$

An NFA is said to recognize an input string if, starting in the start state, there exists a set of moves based on the input that takes us into one of the final states

Non-deterministic Versus Deterministic Finite State Automata

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A deterministic finite state automaton (DFA) is one in which

- There are no ϵ-moves
- There is a unique move from any state r on an input symbol a, ie, if $m(r, a) = s$ and $m(r, a) = t$, then $s = t$
- There is a transition out of every state r on every input symbol a

Non-deterministic Versus Deterministic Finite State Automata

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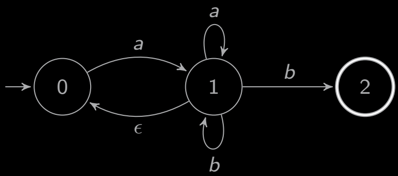
Example: an NFA N that recognizes the language described by $a(a|b)*b$ over the alphabet $\{a, b\}$



Formally, $N = (\Sigma, S, s_0, F, M)$ where $\Sigma = \{a, b\}$, $S = \{0, 1, 2\}$, $s_0 = 0$, $F = \{2\}$, and M is

r	a	$m(r, a)$
0	a	1
1	ϵ	0
1	a	1
1	b	1
1	b	2

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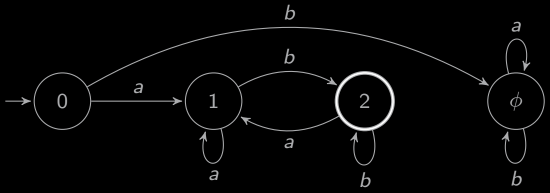
Example: a DFA D that recognizes the language described by $a(a|b)*b$ over the alphabet $\{a, b\}$



Formally, $D = (\Sigma, S, s_0, F, M)$ where $\Sigma = \{a, b\}$, $S = \{0, 1, 2, \phi\}$, $s_0 = 0$, $F = \{2\}$, and M is

r	a	$m(r, a)$
0	a	1
0	b	ϕ
1	a	1
1	b	2
2	a	1
2	b	2
ϕ	a, b	ϕ

Regular Expressions to NFA

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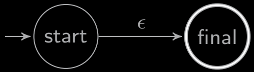
Given any regular expression r, we can construct an NFA N that recognizes the same language, ie, $L(N) = L(r)$

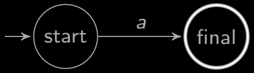
Regular Expressions to NFA · Thompson's Construction (Epsilon Rule)

NFA N_r for recognizing $L(r = \epsilon)$



Regular Expressions to NFA · Thompson's Construction (Singleton Rule)

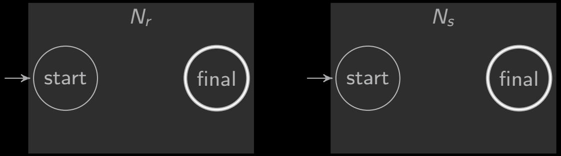
NFA N_r for recognizing $L(r = a)$



Regular Expressions to NFA · Thompson's Construction (Concatenation Rule)

Regular Expressions to NFA · Thompson's Construction (Concatenation Rule)

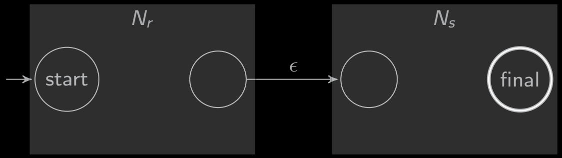
NFAs N_r and N_s for recognizing $L(r)$ and $L(s)$



Regular Expressions to NFA · Thompson's Construction (Concatenation Rule)

NFAs N_r and N_s for recognizing $L(r)$ and $L(s)$

NFA N_{rs} for recognizing $L(rs)$



Regular Expressions to NFA · Thompson's Construction (Alternation Rule)

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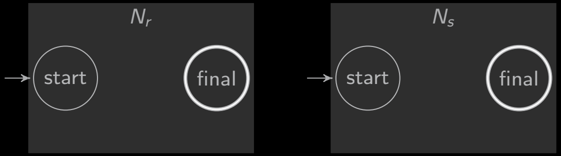
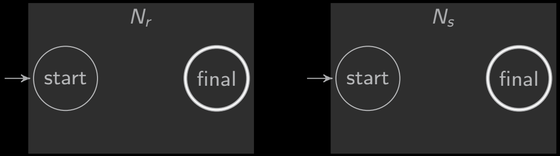
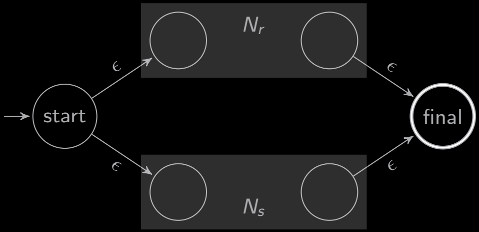
NFAs N_r and N_s for recognizing $L(r)$ and $L(s)$



NFAs N_r and N_s for recognizing $L(r)$ and $L(s)$



NFA $N_{r|s}$ for recognizing $L(r|s)$



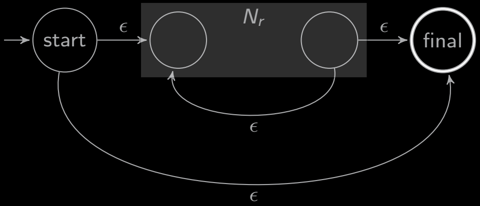
Regular Expressions to NFA · Thompson's Construction (Kleene Closure Rule)

NFAs N_r for recognizing $L(r)$

Regular Expressions to NFA · Thompson's Construction (Kleene Closure Rule)

NFAs N_r for recognizing $L(r)$

NFA N_{r*} for recognizing $L(r*)$



Regular Expressions to NFA · Thompson's Construction (Grouping Rule)

NFA N_r for recognizing $L(r)$ also recognizes $L((r))$

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Example (NFA for $(a|b)a*b$)

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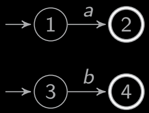
Using the singleton rule, we get the NFAs N_a and N_b for recognizing a and b as



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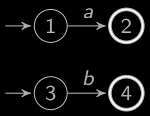
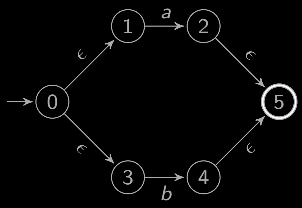


Using the alternation and grouping rules, we get the NFA $N_{(a|b)}$ for recognizing $(a|b)$ as



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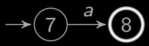
Using the singleton rule, we get the NFAs N_a for recognizing the second instance of a as



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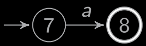
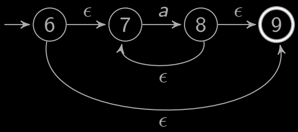


Using the Kleene closure rule, we get the NFA N_{a*} for recognizing $a*$ as



Regular Expressions to NFA

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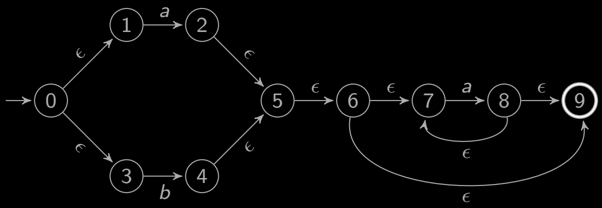
Using the concatenation rule, we get the NFA $N_{(a|b)a*}$ for recognizing $(a|b)a*$



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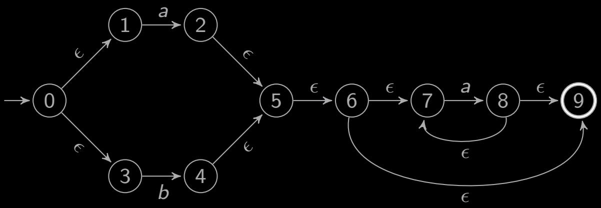
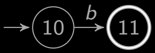


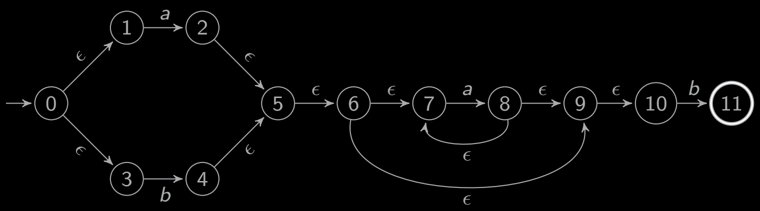
Using the singleton rule, we get the NFAs N_b for recognizing the second instance of b as



Regular Expressions to NFA

Regular Expressions to NFA

Finally, using the concatenation rule, we get the NFA $N_{(a|b)a*b}$ for recognizing $(a|b)a*b$ as



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$$\epsilon\text{-closure}(S) = \bigcup_{s \in S} \epsilon\text{-closure}(s)$$

NFA to DFA · ϵ-closure(S)

Input: a set of states S
Output: ϵ-closure(S)

$P \leftarrow$ Stack(S)
$C \leftarrow$ Set(S)
while not P.isEmpty() **do**
    $r \leftarrow P$.pop()
    for $s \in m(r, \epsilon)$ **do**
        if $s \notin C$ **then**
            P.push(s)
            C.add(s)
        end if
    end for
end while
return C

NFA to DFA · ϵ-closure(s)

NFA to DFA · ϵ-closure(s)

Input: a state s
Output: ϵ-closure(s)

$S \leftarrow \mathrm{Set}(s)$
return ϵ-closure(S)

NFA to DFA · Subset Construction

Input: an NFA $N = (\Sigma, S, s_0, M, F)$
Output: an equivalent DFA $D = (\Sigma, S_D, s_{D0}, M_D, F_D)$

$s_{D0} \leftarrow \epsilon\text{-closure}(s_0)$
$S_D \leftarrow \text{Set}(s_{D0})$
$M_D \leftarrow Moves()$
$stk \leftarrow \text{Stack}(s_{D0})$
$i \leftarrow 1$
while not $stk.\text{isEmpty}()$ **do**
$r \leftarrow stk.\text{pop}()$
for $a \in \Sigma$ **do**
$s_{Di} \leftarrow \epsilon\text{-closure}(\cup_{r' \in r} m(r', a))$
if $s_{Di} \neq \{\}$ **then**
if $s_{Di} \notin S_D$ **then**
$S_D.\text{add}(s_{Di})$
$stk.\text{push}(s_{Di})$
$i \leftarrow i + 1$
$M_D.\text{add}((r, a) \rightarrow s_{Di})$
else
$M_D.\text{add}((r, a) \rightarrow s_j)$, where $s_j \in S_D$ such that $s_j = s_{Di}$
end if
else
$S_D.\text{add}(\phi)$; $M_D.\text{add}((r, a) \rightarrow \phi)$; and $M_D.\text{add}((\phi, a) \rightarrow \phi)$
end if
end for
end while
$F_D \leftarrow \text{Set}()$
for $s_D \in S_D$ **do**
for $s \in s_D$ **do**
if $s \in F$ **then**
$F_D.\text{add}(s_D)$
end if
end for
end for
return $D = (\Sigma, S_D, s_{D0}, M_D, F_D)$

NFA to DFA

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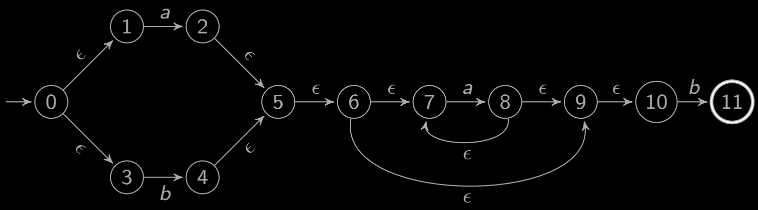
Example (DFA for $N_{(a|b)a*b}$)



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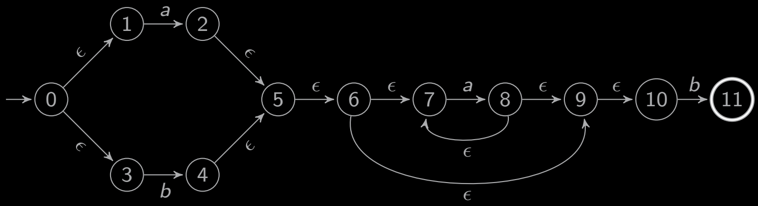

r	a	$m(r, a)$

NFA to DFA

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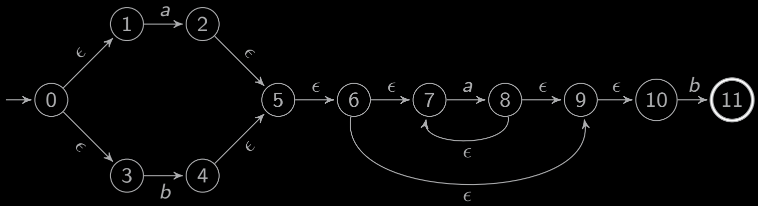

r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)		

NFA to DFA

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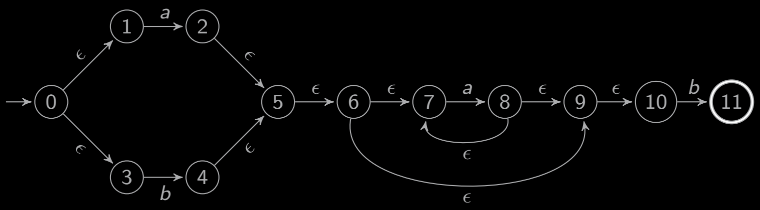

r	a	$m(r, a)$
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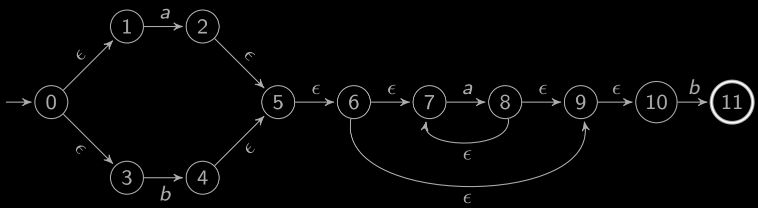
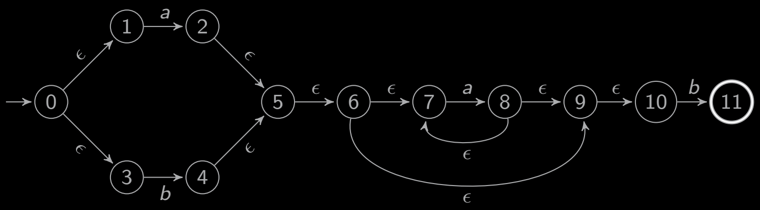


r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)	a	$\{2, 5, 6, 7, 9, 10\} = 1$

NFA to DFA

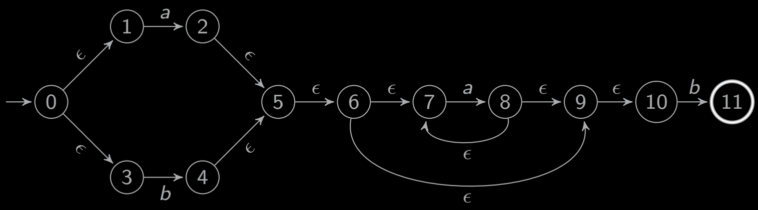
Example (DFA for $N_{(a|b)a*b}$)



r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)	a	$\{2, 5, 6, 7, 9, 10\} = 1$
0		

NFA to DFA

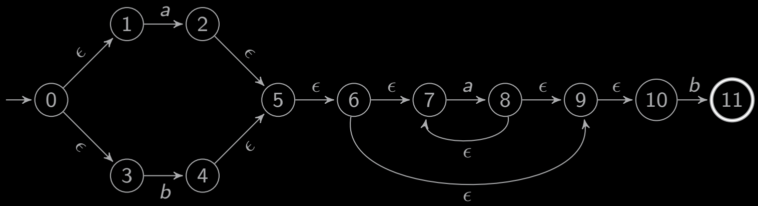
Example (DFA for $N_{(a|b)a*b}$)



r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)	a	$\{2, 5, 6, 7, 9, 10\} = 1$
0	b	

NFA to DFA

Example (DFA for $N_{(a|b)a*b}$)



r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)	a	$\{2, 5, 6, 7, 9, 10\} = 1$
0	b	$\{4, 5, 6, 7, 9, 10\} = 2$

NFA to DFA

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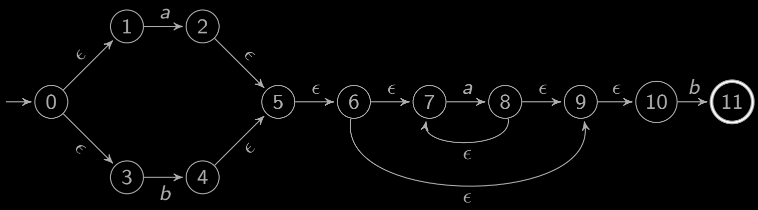

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1		

NFA to DFA

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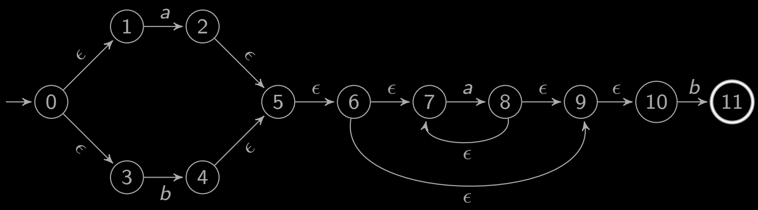

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NFA to DFA

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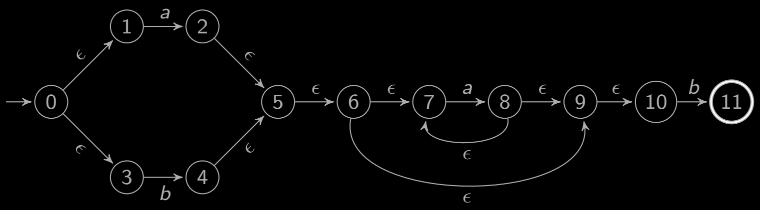

r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)	a	$\{2, 5, 6, 7, 9, 10\} = 1$
0	b	$\{4, 5, 6, 7, 9, 10\} = 2$
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NFA to DFA

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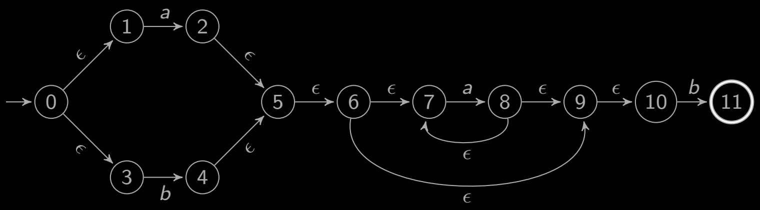

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NFA to DFA

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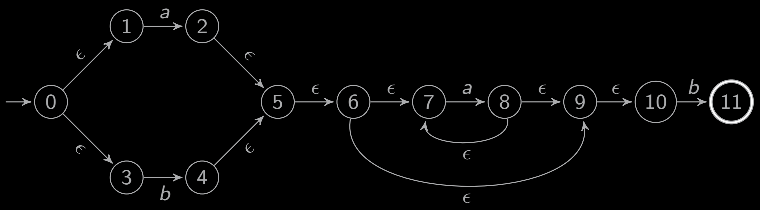

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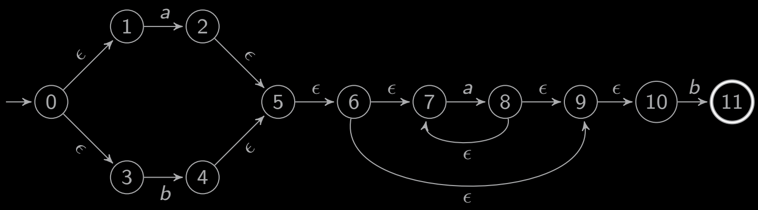
1	b	

NFA to DFA

Example (DFA for $N_{(a|b)a*b}$)



r	a	$m(r,a)$
$\{0,1,3\} = 0$ (start state)	a	$\{2,5,6,7,9,10\} = 1$
0	b	$\{4,5,6,7,9,10\} = 2$
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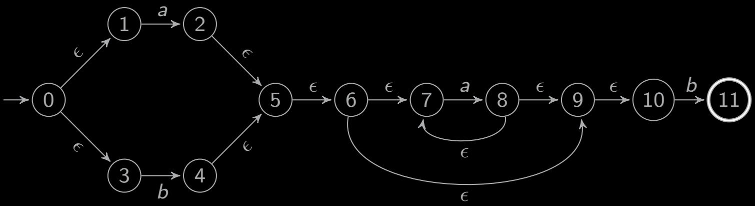
1	b	$\{11\} = 4$ (final state)

NFA to DFA

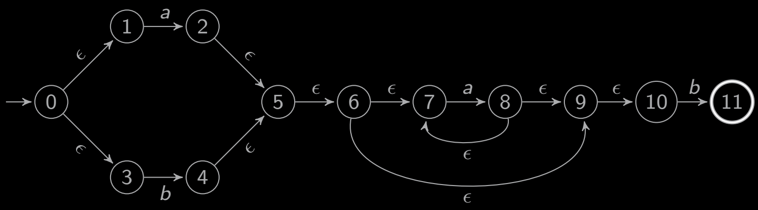
Example (DFA for $N_{(a|b)a*b}$)



r	a	$m(r, a)$
$\{0, 1, 3\} = 0$ (start state)	a	$\{2, 5, 6, 7, 9, 10\} = 1$
0	b	$\{4, 5, 6, 7, 9, 10\} = 2$
1	a	$\{7, 8, 9, 10\} = 3$
1	b	$\{11\} = 4$ (final state)
2		

NFA to DFA

Example (DFA for $N_{(a|b)a*b}$)



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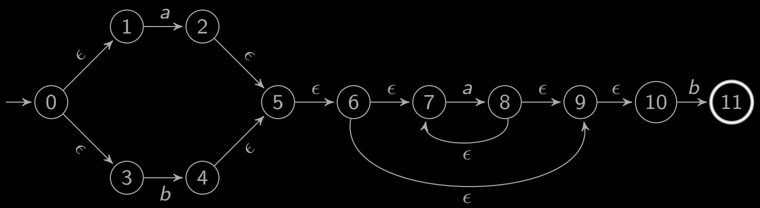

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2	a	3

NFA to DFA

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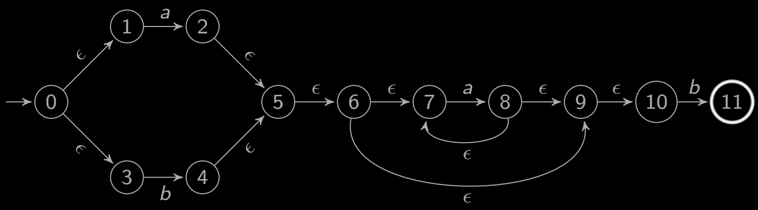

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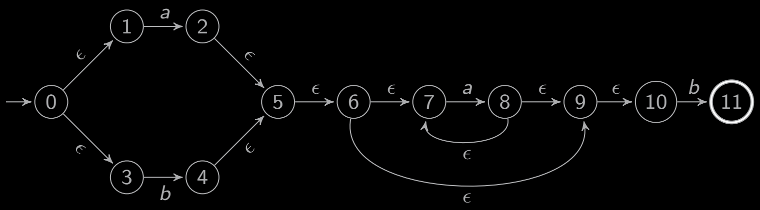
2	a	3
2		

NFA to DFA

Example (DFA for $N_{(a|b)a*b}$)



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2	a	3
2	b	

NFA to DFA

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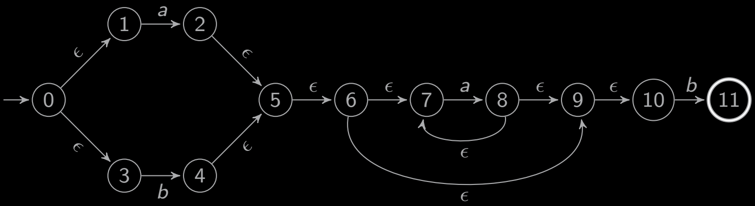

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NFA to DFA

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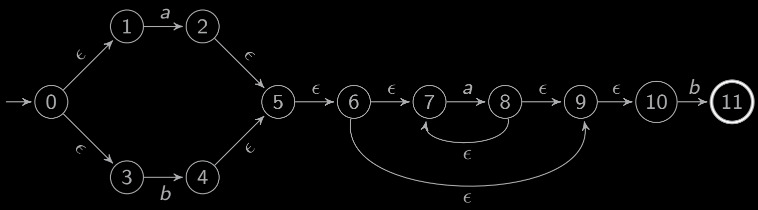

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3		

NFA to DFA

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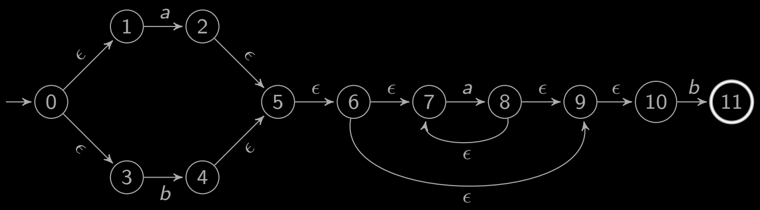

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2	b	4
3	a	

NFA to DFA

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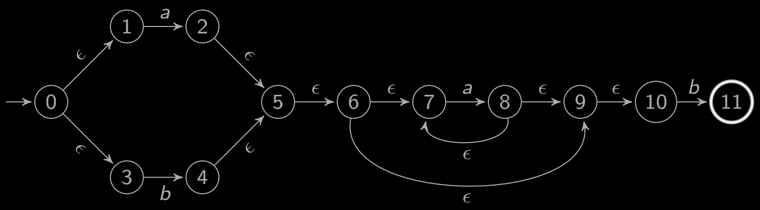

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NFA to DFA

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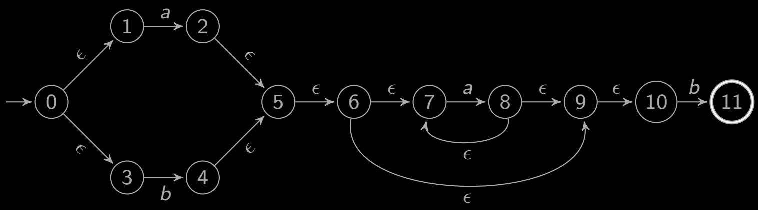

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3		

NFA to DFA

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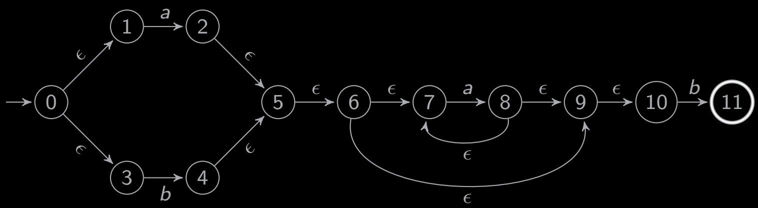

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2	b	4
3	a	3
3	b	

NFA to DFA

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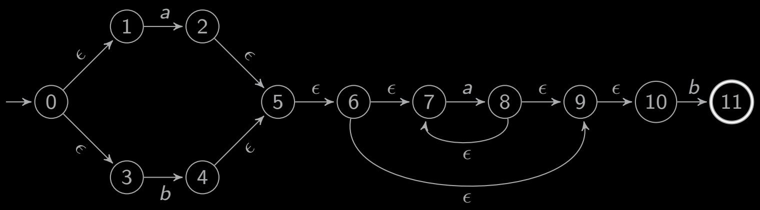

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NFA to DFA

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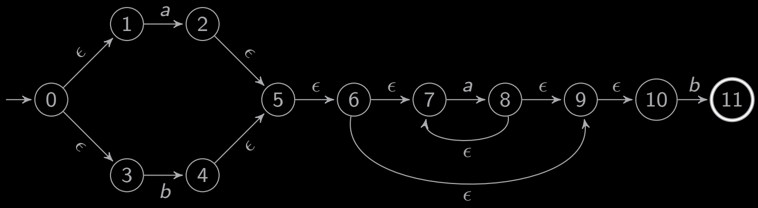

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3	b	4
4		

NFA to DFA

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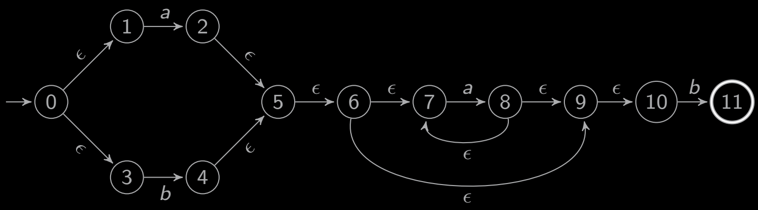

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4	a, b	

NFA to DFA

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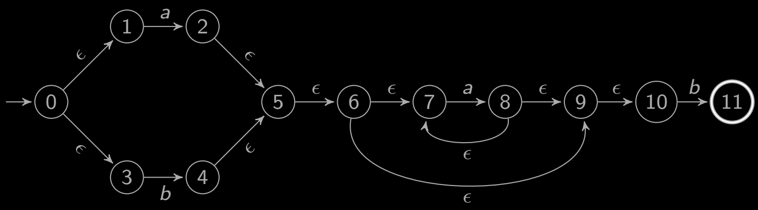

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4	a, b	ϕ

NFA to DFA

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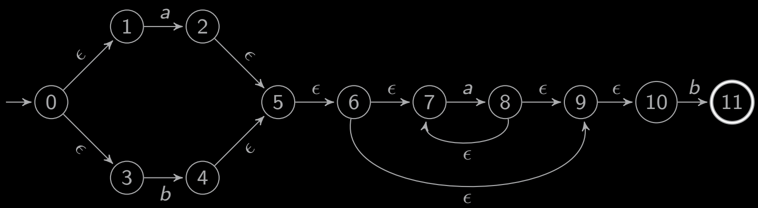

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ϕ		

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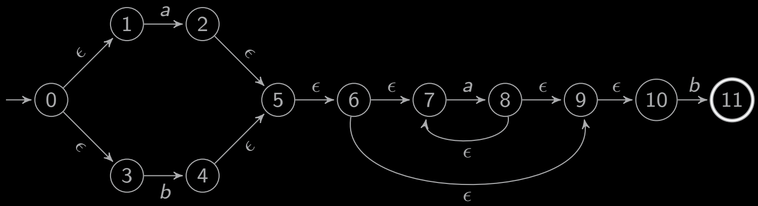

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3	a	3
3	b	4
4	a, b	ϕ
ϕ	a, b	

NFA to DFA

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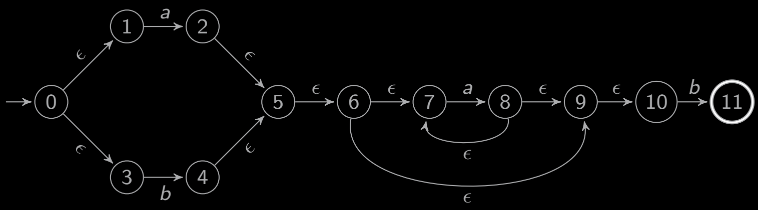

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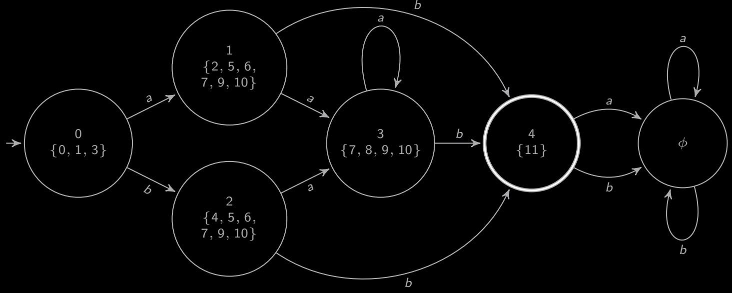
4	a, b	ϕ
ϕ	a, b	ϕ

NFA to DFA

NFA to DFA



Minimal DFA

Minimal DFA

To obtain a smaller but equivalent DFA, we partition the states such that the states in the new DFA are subsets of the states in the original DFA

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The initial partition contains two subsets: the non-final states and the final states

We make sure that from each subset, on each input symbol, we transition into an identical subset; otherwise, we split the subset

Minimal DFA · Partitioning

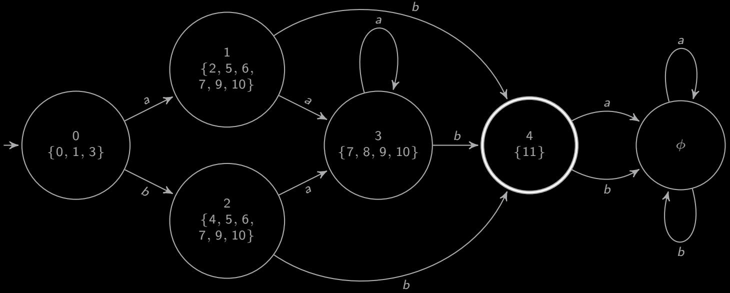
Input: a DFA $D = (\Sigma, S, s_0, M, F)$
Output: a partition of S

$\mathcal{P} \leftarrow \{S - F, F\}$
while splitting occurs **do**
  for $Q \in \mathcal{P}$ **do**
    if Q.size() > 1 **then**
      for $a \in \Sigma$ **do**
        $r \leftarrow$ a state chosen from Q
        $T \leftarrow$ the subset in the $\mathcal{P}$ containing $m(r, a)$
        $Q_1 \leftarrow \{s \in Q | m(s, a) \in T\}$
        $Q_2 \leftarrow \{s \in Q | m(s, a) \notin T\}$
        if $Q_2 \neq \{\}$ **then**
          replace Q in $\mathcal{P}$ by Q_1 and Q_2
          break
        end if
      end for
    end if
  end for
end while
return $\mathcal{P}$

Minimal DFA

Minimal DFA

Example (minimal DFA for $D_{(a|b)a*b}$)



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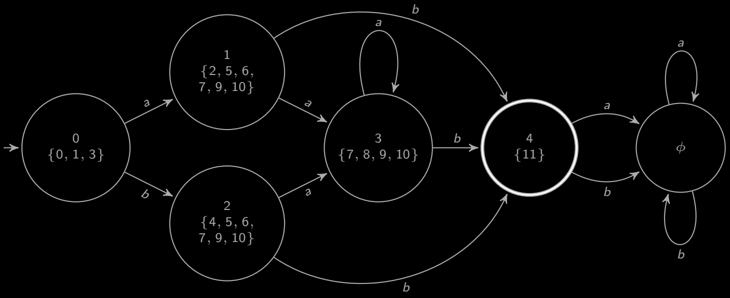

Initial partition $\mathcal{P} = \{\{0, 1, 2, 3, \phi\}, \{4\}\}$

Minimal DFA

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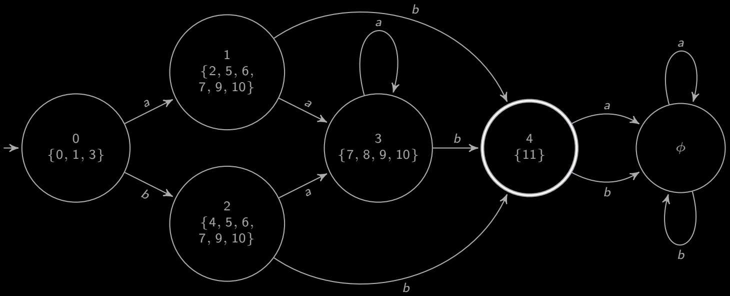

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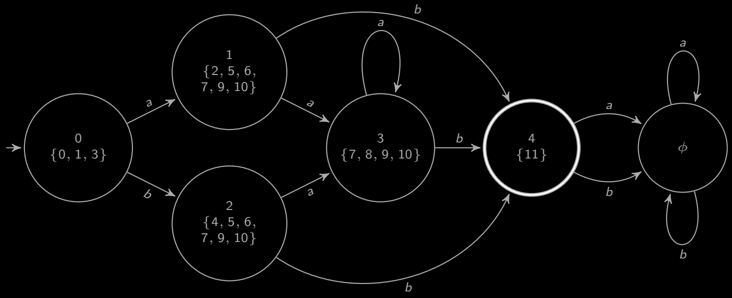
The input symbol a does not split the subset $Q = \{0, 1, 2, 3, \phi\}$

Minimal DFA

Example (minimal DFA for $D_{(a|b)a*b}$)



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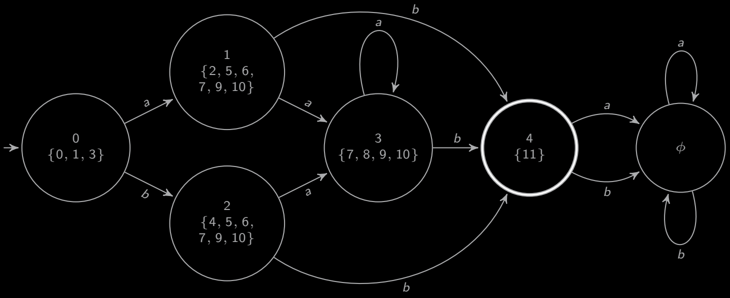
The input symbol a does not split the subset $Q = \{0, 1, 2, 3, \phi\}$

The input symbol b splits the subset $Q = \{0, 1, 2, 3, \phi\}$ into $Q_1 = \{0, \phi\}$ and $Q_2 = \{1, 2, 3\}$

Minimal DFA

Example (minimal DFA for $D_{(a|b)a*b}$)



Initial partition $\mathcal{P} = \{\{0, 1, 2, 3, \phi\}, \{4\}\}$

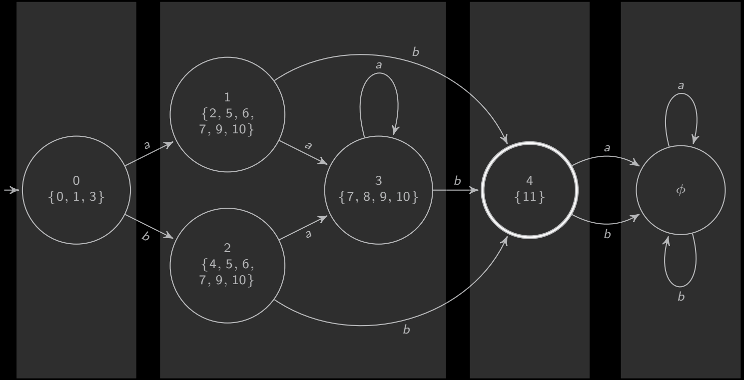
The input symbol a does not split the subset $Q = \{0, 1, 2, 3, \phi\}$

The input symbol b splits the subset $Q = \{0, 1, 2, 3, \phi\}$ into $Q_1 = \{0, \phi\}$ and $Q_2 = \{1, 2, 3\}$

The new and final partition is $\mathcal{P} = \{\{0\}, \{1, 2, 3\}, \{4\}, \{\phi\}\}$

Minimal DFA

Minimal DFA



Minimal DFA