# Welcome to

# Theory of Computation

CS 420 / CS 620

UMass Boston Computer Science

Instructors: Stephen Chang and Holly DeBlois

Fall 2025

Today's Theme:
What's this course about?

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# Theory of Computation

What's this?

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# *Interlude:* Lecture Logistics

- *I expect:* **lecture** to be **interactive**
  - **Participation** is a part of your **grade**
  - Also, **it's the best way to learn!**
- *I may:* **call** on **students randomly**
  - It's **ok to be wrong in class**! – will not affect your grade
  - Also, **it's the best way to learn!**
- *Please:* **tell me your name** before **speaking**
  - Sorry in advance if I get it wrong
  - Also, **it's the best way for me to learn!**

# Welcome to

# Theory of Computation

How would you define this?

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# Computation Is … (via examples)

- 1 + 1 = ??
- = 2

… some **basic** <u>**definitions**</u> and <u>**assumptions**</u> (**“axioms”**),
e.g., define “Numbers” to be: 0, 1, 2, 3, … …

- 11 + 11 = ??
- = 22

… and <u>**rules that use**</u> the **definitions** and **axioms** (**“algorithm”**),
e.g., grade school arithmetic

- 9999999999 + 9999999999 = ??
- = 19999999998

**Computation rules** can be <u>**executed**</u> by
**hand**, or by **machine** / **automaton**

- 1 +1 = ??
- = 10

(binary)

There are <u>many</u> **possible definitions**
(i.e., **models**) of **computation**

(hint)

# Computation Is … Programs!

Every **programming language** <u>is</u> a **model of computation**

**different???**

If they are <u>different</u>:
how can we know?

**Or same???**

If they are the <u>same</u>:
is there a common
model for all?

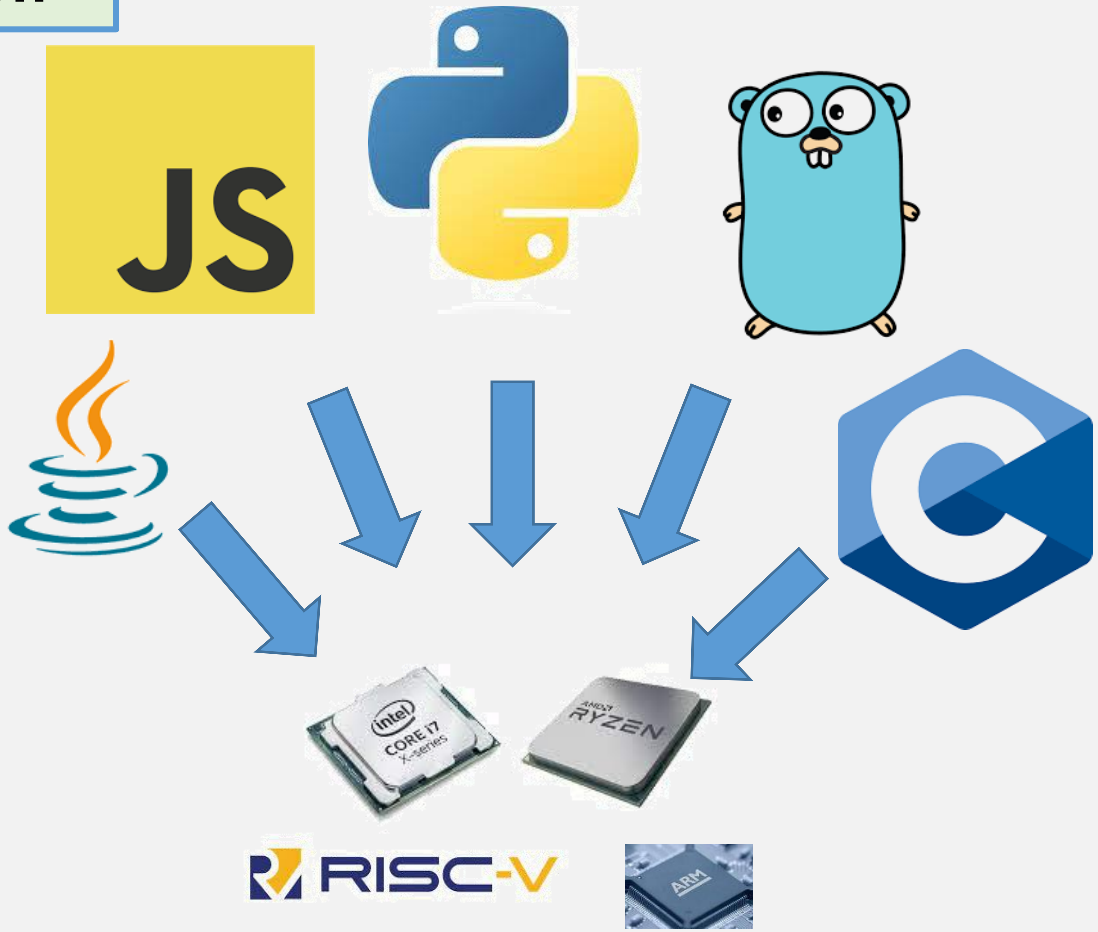
```
def bigger(x):
    if x > 0:
        return x + 1
    else:
        return x – 1

print( bigger(10) )
```

???

→ 11

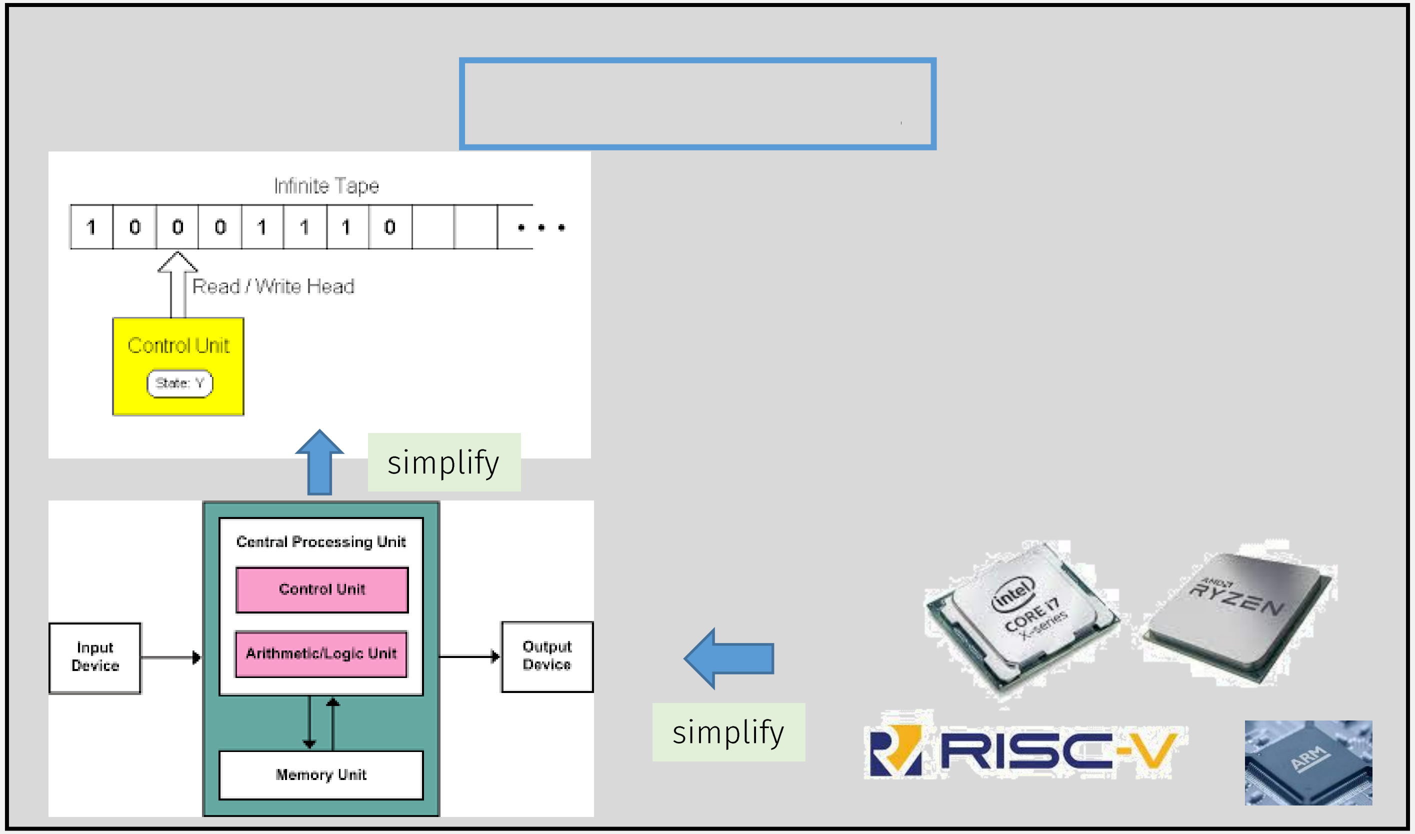
You <u>already</u> use
**models of computation!**
Every time you
reason about code!



# This semester, we will …

1. Define and study **models of computation**
   - **models** will be *as simple as possible* (to make them easier to study)

# Models of Computation (spoiler alert!)



# This semester, we will …

1. Define and study **models of computation**
   - **models** will be *as simple as possible* (to make them easier to study)

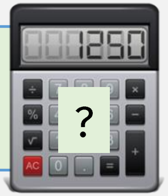
2. Compare and contrast models of computation
   - which “programs” are *included* by a model
   - which “programs” are *excluded* by a model
   - *overlap* between models?

# Models of Computation

Turing Machines

**$Q_1$**: Are there computational models … other than **Turing Machines?**

**$Q_2$**: Are there computational models … "*weaker*" than **Turing Machines?**




**$Q_3$**: Are there computational models … "*more powerful*" than **Turing Machines?**



RISC-V

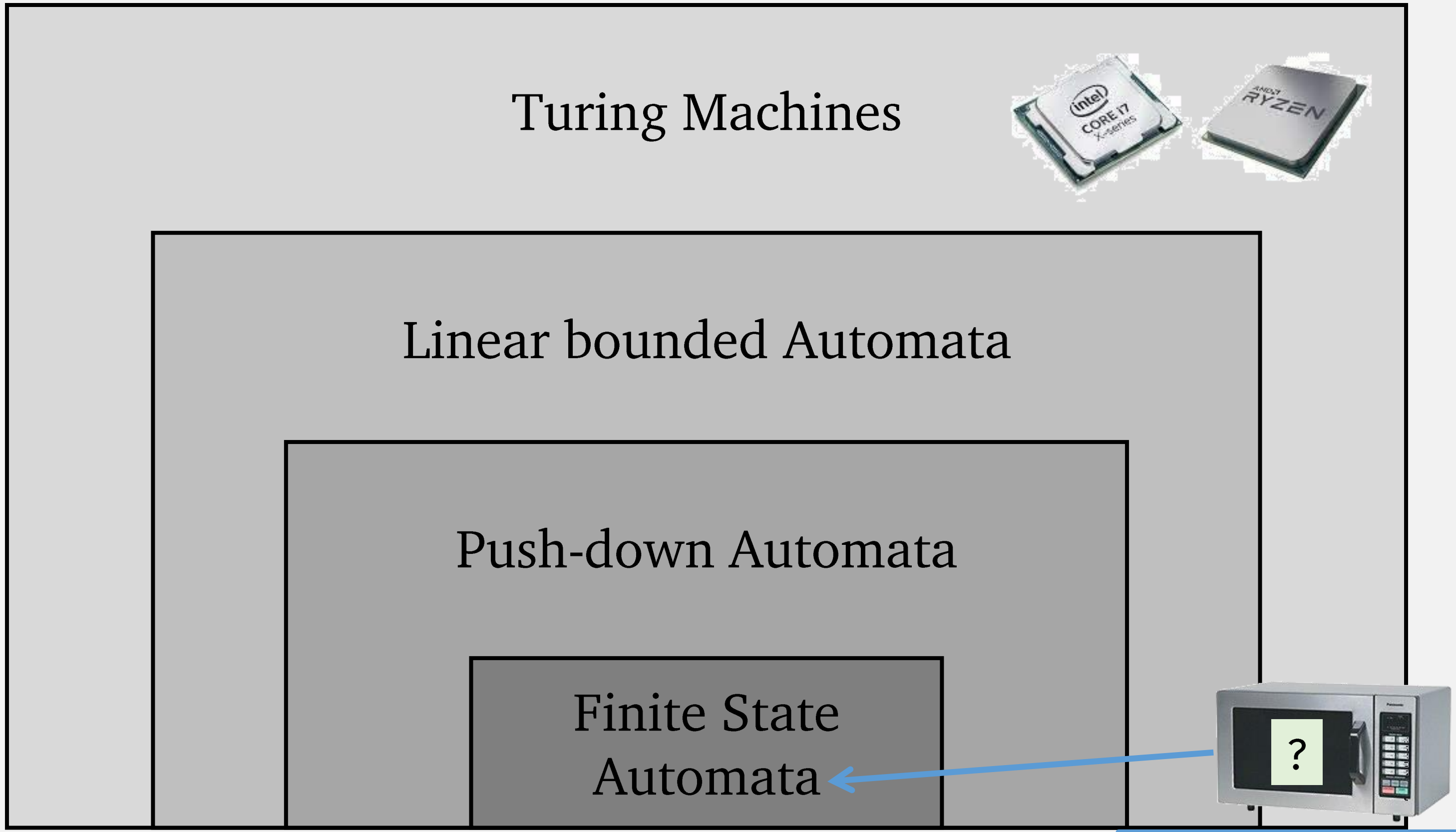

**$Q_4$**: What does "*weaker*" or "*more powerful*" even mean?!

**A**: Yes, yes, yes, and … stay tuned!

# Models of Computation Hierarchy

... and get to here ...

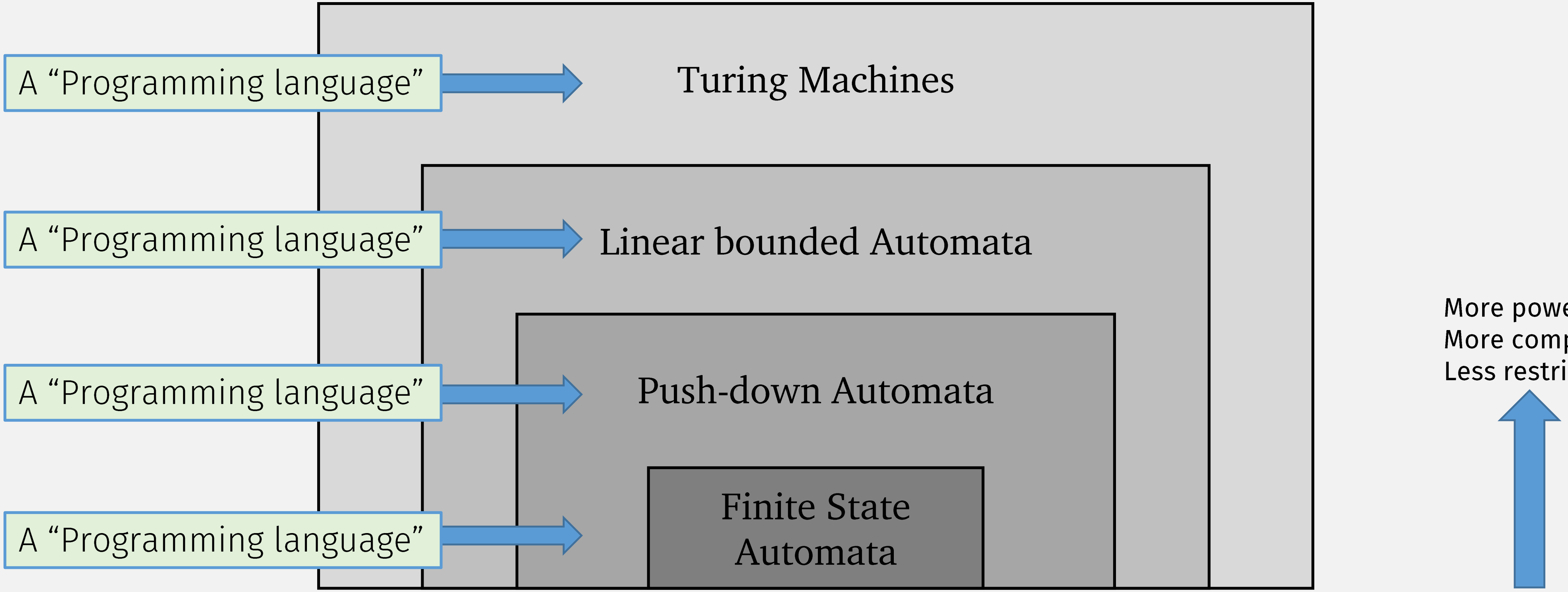
... and also look at what's out here???


We'll start here ...

More powerful
More complex
Less restricted

# *But remember* … Computation = Programs!




*Helpful* analogy *for this course:*

- a **set** of machines / computational model (a rectangle) ~ a **Programming Language!**
- a **single** machine (one thing in a rectangle) ~ a **Program!**

What's this? ☑

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**"Theory" = math**

(This is a math course!)

(But programming is math too!)

# Programming Is (What) Math?

Math(ematical) logic!

```
def bigger(x):
    if x > 0:
        return x + 1
    else:
        return x - 1

print( bigger(10) )
```

???

→ 11

How did you figure out the answer?

(But programming is math too!)

# Programming = Mathematical logic!

- “logic is the foundation of all computer programming”
  - https://www.technokids.com/blog/programming/its-easy-to-improve-logical-thinking-with-programming/

- “logic is the fundamental key to becoming a good developer”
  - https://www.geeksforgeeks.org/i-cant-use-logic-in-programming-what-should-i-do/

- “Analytical skill and logical reasoning are prerequisites of programming because coding is effectively logical problem solving at its core”
  - https://levelup.gitconnected.com/the-secret-weapon-of-great-software-engineers-22d57f427937

(Studying logic, i.e., this class, will make you a better programmer!)

# Programming = Mathematical logic!

**Programming** Concepts

- Functions
- Variables
- If-then
- Recursion
- Strings
- **Sets** (and other data structures)

**Math**(ematical Logic) Concepts

- Functions
- Variables
- If-then (implication)
- Recursion
- Strings
- **Sets** (and other groupings of data)

(Studying logic, i.e., this class,  will make you a better programmer!)

# This semester, we will …

1. Define and study **models of computation**
   - models will be *as simple as possible* (to make them easier to study)

2. Compare and contrast models of computation
   - which “programs” are *included* by a model
   - which “programs” are *excluded* by a model
   - *overlap* between models?

3. Prove things about the models

# Reasoning About Code <u>is</u> ~~Math~~ Proof

```
def no_div0(x):
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1
    else:
        return 1 / 0

print( no_div0(10) ) ???
```

→ 11

Can this function ever throw `ZeroDivisionError`? No!

How did you figure out the answer? You used the **Python model of computation** to **predict the program's behavior**

**You did a proof!**

# A (Mathematical) Theory Is …

## Mathematical theory

From Wikipedia, the free encyclopedia

A **mathematical theory** is a mathematical model of a branch of mathematics that is based on a set of axioms. It can also simultaneously be a body of knowledge (e.g., based on known axioms and definitions), and so in this sense can refer to an area of mathematical research within the established framework.[1][2]

Explanatory depth is one of the most significant theoretical virtues in mathematics. For example, set theory has the ability to systematize and explain number theory and geometry/analysis. Despite the widely logical necessity (and self-evidence) of arithmetic truths such as 1<3, 2+2=4, 6-1=5, and so on, a theory that just postulates an infinite blizzard of such truths would be inadequate. Rather an adequate theory is one in which such truths are derived from explanatorily prior axioms, such as the Peano Axioms or set theoretic axioms, which lie at the foundation of ZFC axiomatic set theory.

The singular accomplishment of axiomatic set theory is its ability to give a foundation for the derivation of the entirety of classical mathematics from a handful of axioms. The reason set theory is so prized is because of its explanatory depth. So a mathematical theory which just postulates an infinity of arithmetic truths without explanatory depth would not be a serious competitor to Peano arithmetic or Zermelo-Fraenkel set theory.[3][4]

… a mathematical model,
i.e., **axioms** and **definitions**, of
some domain, e.g. computers …

… that **explains** (**predicts**)
some real-world phenomena …

… and can **derive** (prove)
additional results (**theorems**) …

# How Mathematics (Proofs) Work

# The “Modus Ponens” Inference Rule

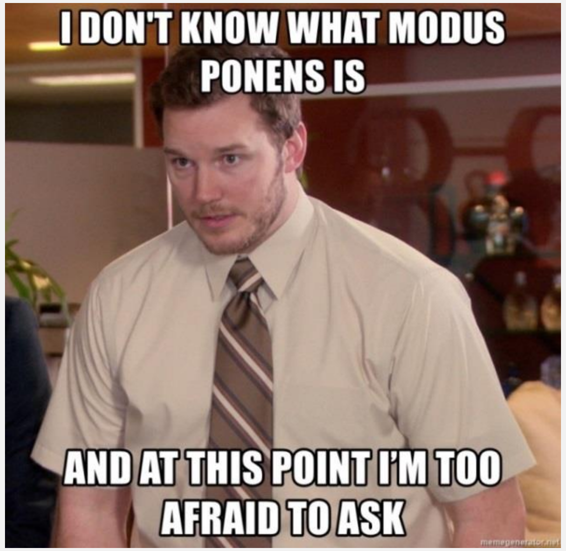
(Precisely Fitting Blocks Together)

**Premises** (if we can show these statements are true)

- If $P$ then $Q$
- $P$ is TRUE

**Conclusion** (then we can say that this is also true)

- $Q$ must also be TRUE



# Kinds of Mathematical Proof

**Deductive Proof**

- *Start with*: <u>known</u> facts and statements
- *Use:* logical **inference rules** (like modus ponens) to **prove** <u>new</u> **facts** and **statements**

# You already do “Proof” when Programming

```
def no_div0(x):
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1
    else:
        return 1 / 0

print( no_div0(10) )
```

???

→ 11

Can this function ever throw `ZeroDivisionError`?

No!

How did you figure out the answer?

You used the **Python model of computation** to **predict the program’s behavior**

(Let’s write it out formally)

**You did a proof!**

# Deductive Proof Example

```
def no_div0(x):        "test expr"
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1    "first branch"
    else:
        return 1 / 0    "second branch"
```

Prove: `no_div0` never throws `ZeroDivisionError`

**Statements / Justifications Table**

Proof: Prior steps are already-proved, can be used to prove later steps!

| Statements | Justifications |
| --- | --- |
| 1. If running "test expr" is `True`, then "first branch" runs | 1. Rules of Python |
| 2. If running "test expr" is `False`, then "second branch" runs | 2. Rules of Python |
| 3. running "test expr" is (always) `True` | 3. Definition of "numbers" |
| 4. "first branch" (always) runs | 4. By steps **1**, **3**, and modus ponens |
| 7. `no_div0` never throws `ZeroDivisionError` | |

Modus Ponens

If we can prove these:

- If $P$ then $Q$
- $P$

Then we've proved:

- $Q$

# Deductive Proof Example

```
def no_div0(x):
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1
    else:
        return 1 / 0   "second branch"
```

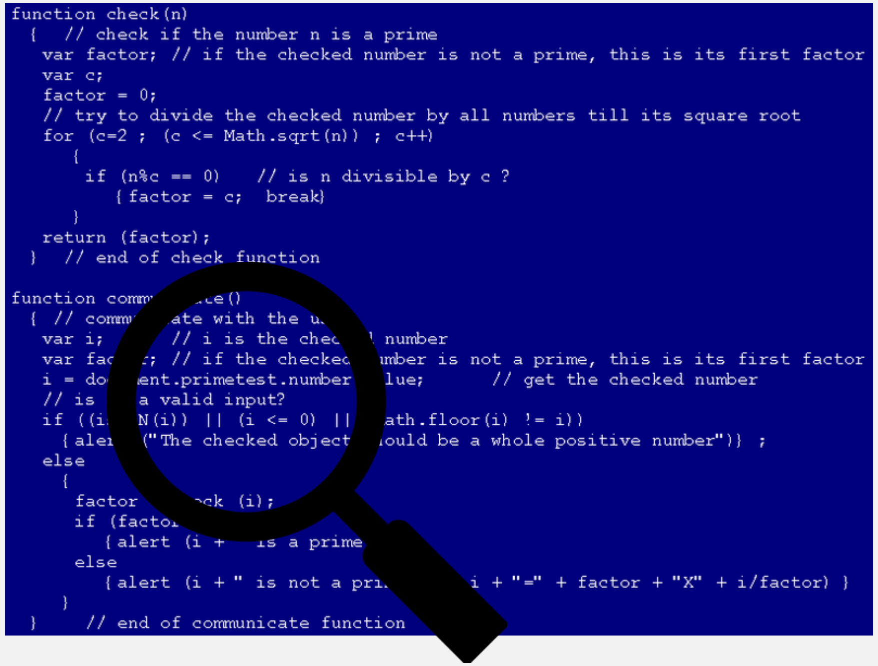
Prove: `no_div0` never throws `ZeroDivisionError`

Proof:

| Statements | Justifications |
| --- | --- |
| 1. If running "test expr" is `True`, then "first branch" runs | 1. Rules of Python |
| 2. If running "test expr" is `False`, then "second branch" runs | 2. Rules of Python |
| 3. running "test expr" is (always) `True` | 3. Definition of "numbers" |
| 4. "first branch" (always) runs | 4. By steps **1**, **3**, and modus ponens |
| 5. "second branch" *never* runs | 5. By step **4**, and Rules of Python??? |
| 6. `no_div0` never runs `1 / 0` | 6. By step **5** |
| 7. `no_div0` never throws `ZeroDivisionError` | 7. By step **6** and Rules of Python??? |

∎

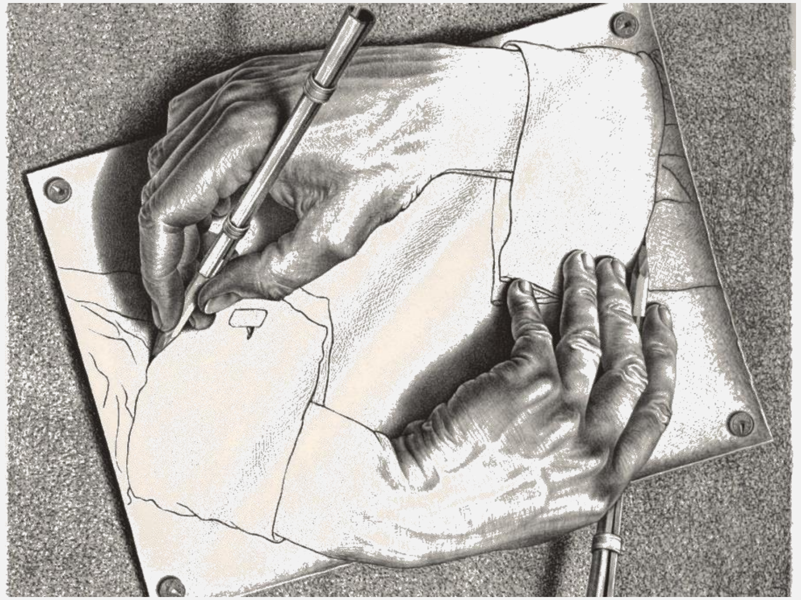
# What else can we prove about programs?





**Proof = prediction** about **program result**
… <u>without</u> **running the program**

# <u>Can</u> we make predictions about computation?



It's tricky: **Trying to predict computation requires computation!**

# Can we make predictions about computation?

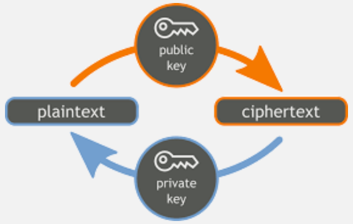
- The **Halting Lemma** says:


- And **Rice's Theorem** says:
  - "all non-trivial, semantic properties of programs are undecidable"

# Knowing What Computers Can't Do is Still Useful!

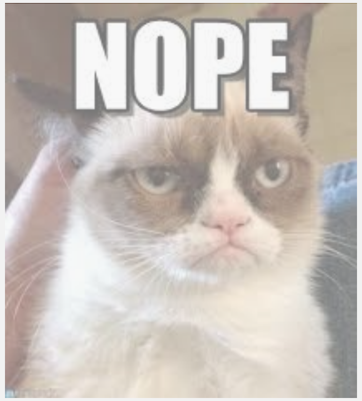
In Cryptography:

- **Perfect secrecy is impossible in practice**
- But with **slightly imperfect secrecy** (i.e., a computationally bounded adversary) **we get:**



# Can we make predictions about computation?

- The **Halting Lemma** says:




- And **Rice's Theorem** says:
  - "all non-trivial, semantic properties of programs are undecidable"

Actually:

- it depends on the computation model!

# Predicting What <u>Some</u> Programs Will Do …

microsoft.com/en-us/research/project/slam/

SLAM is a project for checking that software satisfies critical behavioral properties of the interfaces it uses and to aid software engineers in designing interfaces and software that ensure reliable and correct functioning. Static Driver Verifier is a tool in the Windows Driver Development Kit that uses the SLAM verification engine.

*"Things like even software verification, this has been the Holy Grail of computer science for many decades but now in some very key areas, for example, driver verification we're building tools that can do actual proof about the software and how it works in order to guarantee the reliability."* **Bill Gates, April 18, 2002. Keynote address at WinHec 2002**

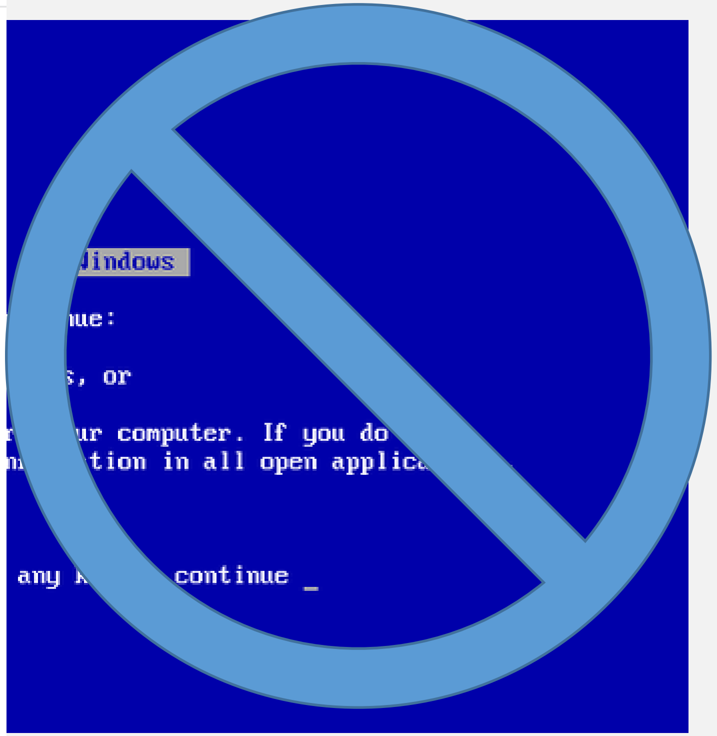
Predicting things about programs … is the **Holy grail of CS**!

Static Driver Verifier Research Platform README

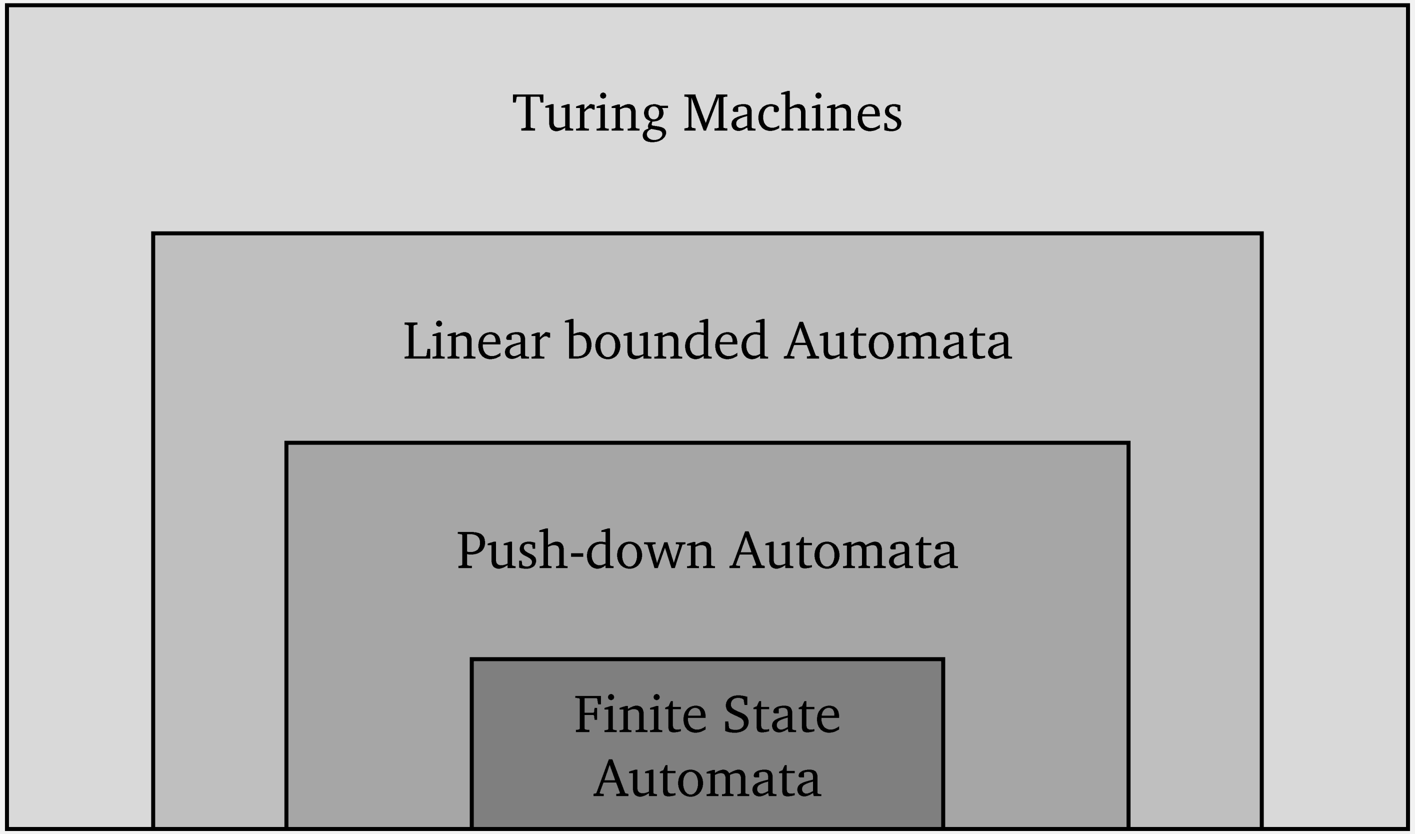
# Overview of Static Driver Verifier Research Platform

Static Driver Verifier (SDV) is a compile-time static verification tool, included in the Windows Driver Kit (WDK). The SDV Research Platform (SDVRP) is an extension to SDV that allows you to adapt SDV to:

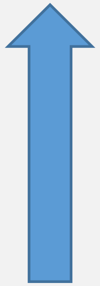
- Support additional frameworks (or APIs) and write custom SLIC rules for this framework.
- Experiment with the model checking step.



# Proofs About Computational Models ... in this class

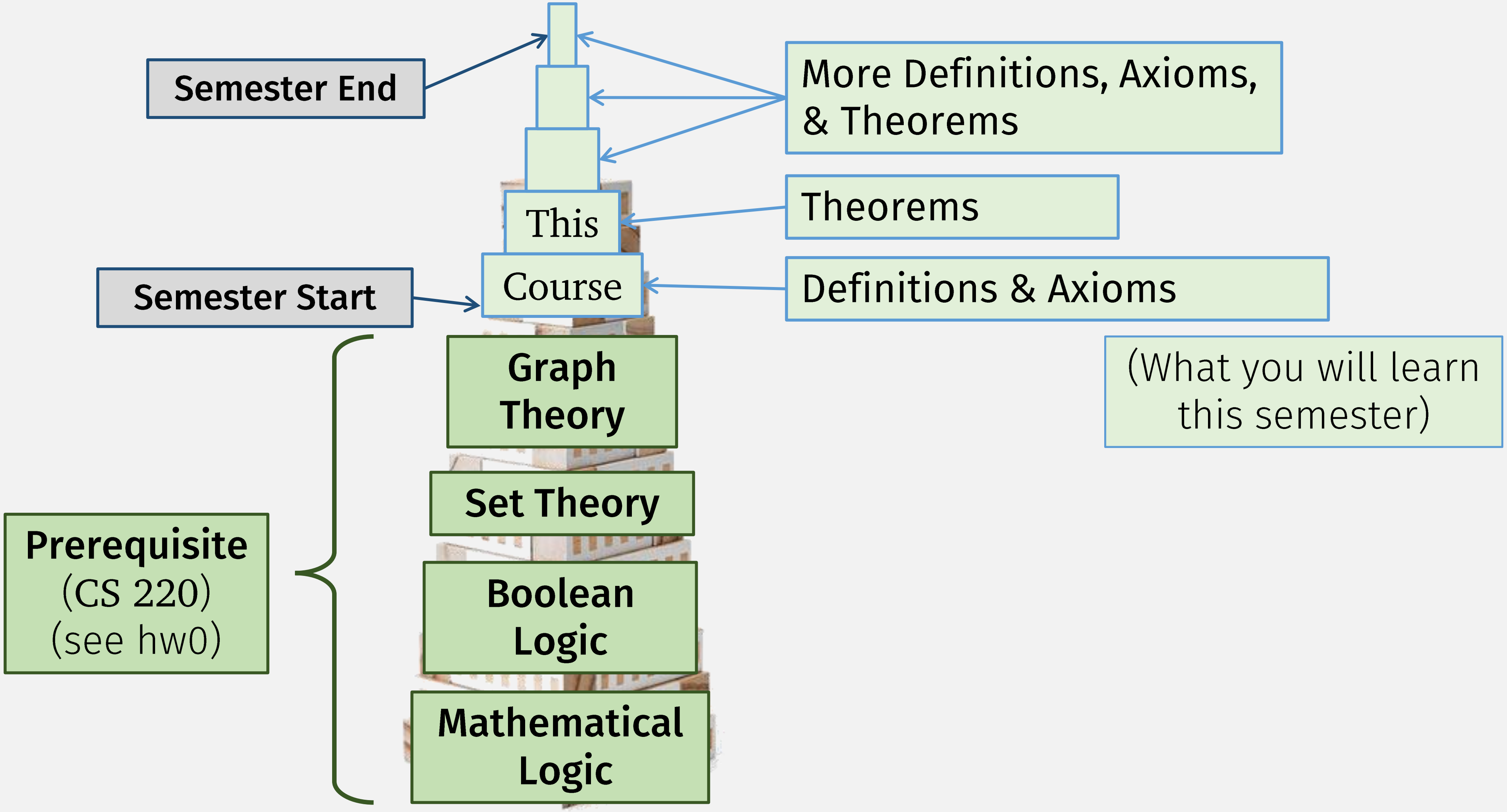



More powerful
More complex
Less restricted



In this class, we will **prove** things about **<u>simple</u> computational models** (not Python ...)

# How This Course Works



# A Word of Advice

Important:
**Do not fall behind in this course**

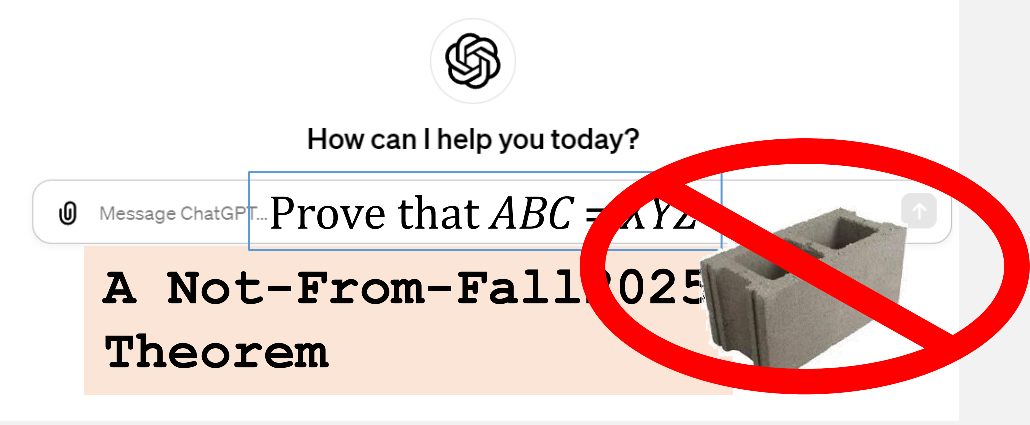
To **prove** a (new) **theorem** ...

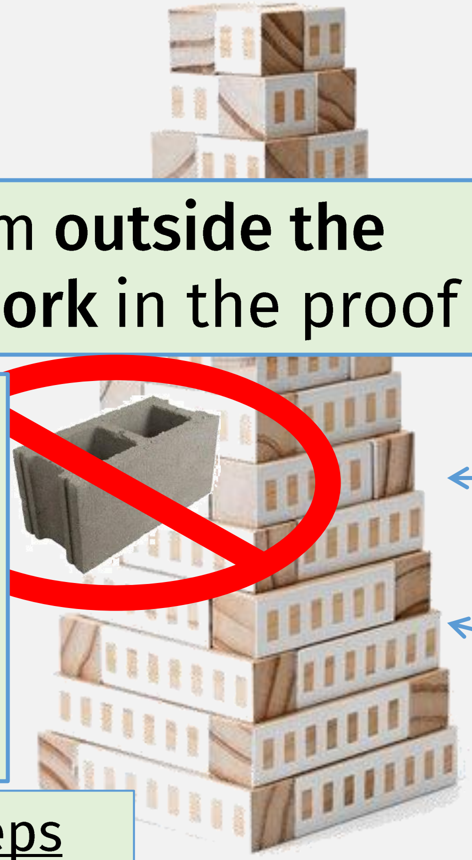
... need to know all **axioms, definitions,** and (previous) **theorems** below it

# Another Word of Advice

HW 1, Problem 1

Prove that $ABC = XYZ$



**"Blocks"** from **outside the course won't work** in the proof

<u>Remember</u>:
**Preciseness in proofs** (just like in programming) **is critical**
(Proofs must connect facts from <u>this course</u> <u>exactly</u>)

... can be used to **prove** (new) **theorems** in <u>this course</u>

<u>Only</u> **axioms, definitions,** and **theorems** from <u>this course</u> ...

HW problems are *graded* on precise <u>steps</u> in the proof, <u>not</u> on the final theorem itself!

# Textbooks

- Sipser. *Intro to Theory of Computation*, 3rd ed.
- Hopcroft, Motwani, Ullman. *Intro to Automata Theory, Languages, and Computation*, 3rd ed.

- **Slides** (posted) and **lecture** will try to be **self-contained**,
- BUT, **students who read the book earn higher grades**

All course info available on (joint) web sites:
- `cs.umb.edu/~stchang/cs620/f25`
- `cs.umb.edu/~stchang/cs420/f25`
- `cs.umb.edu/~hdeblois/cs420/f25/`

# How to Do Well in this Course

- Learn the “ building blocks”
  - I.e., **axioms, definitions,** and **theorems**
- **To solve a problem** (prove a new theorem) **…**
  **… think about how to** (precisely) combine **existing “blocks”**
- **HW problems graded on** steps **to the answer** (not final theorem)
- Don’t Fall Behind!
  - **Start HW Early** (HW 0 due Monday 9/8 12pm EST noon)
- Participate and Engage
  - Lecture
  - Office Hours
  - Message Boards (piazza)

# Grading

- **HW**: 80%
  - Weekly: In / Out Monday
  - Approx. 12 assignments
  - Lowest grade dropped
- **Participation**: 20%
  - Lecture participation, in-class work, office hours, piazza
- No exams

- **A** range: 90-100
- **B** range: 80-90
- **C** range: 70-80
- **D** range: 60-70
- **F**: < 60

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- `cs.umb.edu/~hdeblois/cs420/f25/`

# Late HW

- Is bad … try not to do it please
  - Grades get delayed
  - Can’t discuss solutions
  - You fall behind!

- <u>Late Policy:</u> **3 late days** to use during the semester

# HW Collaboration Policy

**Allowed**

- Discussing HW with classmates (but must cite)
- Using other resources to learn, e.g., `youtube`, other textbooks, …
- Writing up answers <u>on your own</u>, from scratch, in your own words

**Not Allowed**

- Submitting someone else's answer
- Submitting someone else's answer with:
  - variables changed,
  - thesaurus words,
  - or sentences rearranged …
- Using sites like Chegg, CourseHero, Bartleby, Study, ChatGPT, etc.
- Using theorems or definitions <u>not from this course</u>

# Honesty Policy

- 1st offense: zero on problem
- 2nd offense: zero on hw, reported to school
- 3rd offense+: F for course

Regret policy

- If you self-report an honesty violation, you'll only receive a zero on the problem and we move on.

# All Up to Date Course Info

Survey, Schedule, Office Hours, HWs, …

See course website(s):

```
- cs.umb.edu/~stchang/cs620/f25
- cs.umb.edu/~stchang/cs420/f25
- cs.umb.edu/~hdeblois/cs420/f25/
```

# hw0 (pre-req quiz)

(see gradescope)