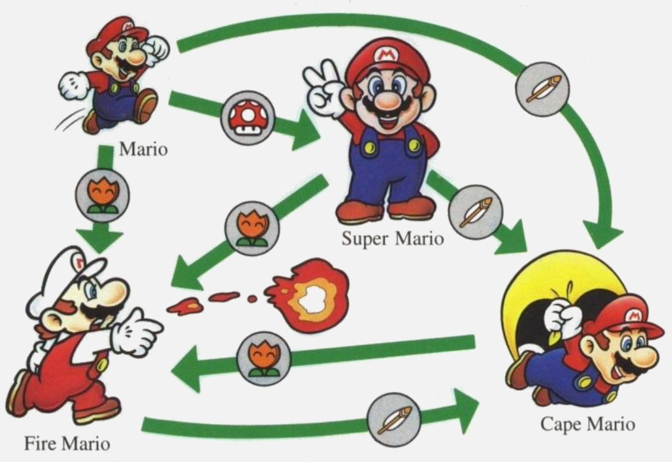
CS420

Finite Automata and Regular Languages

Wed Jan 27, 2021

UMass Boston Computer Science



Programming in Linux: Basics
(15 minute crash course)

Code Demo

- `stdin`, `stdout`
- command line
- command line scripts
- `Makefile`s

A Makefile

comment

Grader Preinstalled langs:
Python, Java, C, C++, JS, Racket

Targets
(see hw for names)

```
setup:  # install your language here (you can probably leave it blank)

run-hw0-stdio:
	racket hello.rkt # this line must start with a tab

run-hw0-alphabet:
	racket alphabet.rkt

run-hw0-powerset:
	racket powerset.rkt

run-hw0-xml:
	racket xml.rkt
```

Commands to run
(these files better exist)

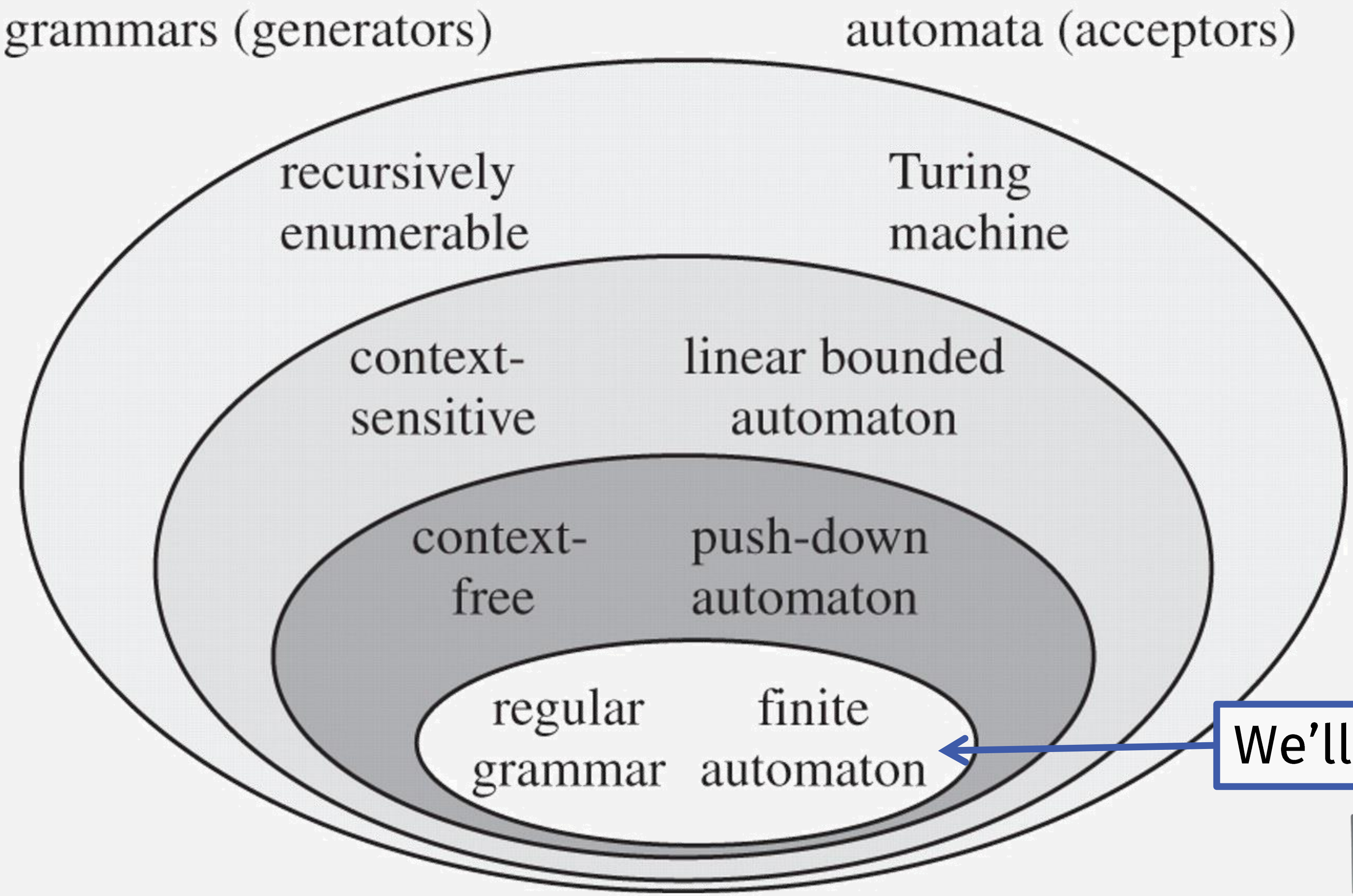
HW 0 Questions?

Last time: The Theory of Computation …

- Creates and studies mathematical models of computers

- In order to:
    - Make predictions about computer programs
    - Explore the limits of computation

Last time: Levels of Computational *Power*



- more complex
- more powerful
- less restricted

We’ll start here

Finite Automata

or

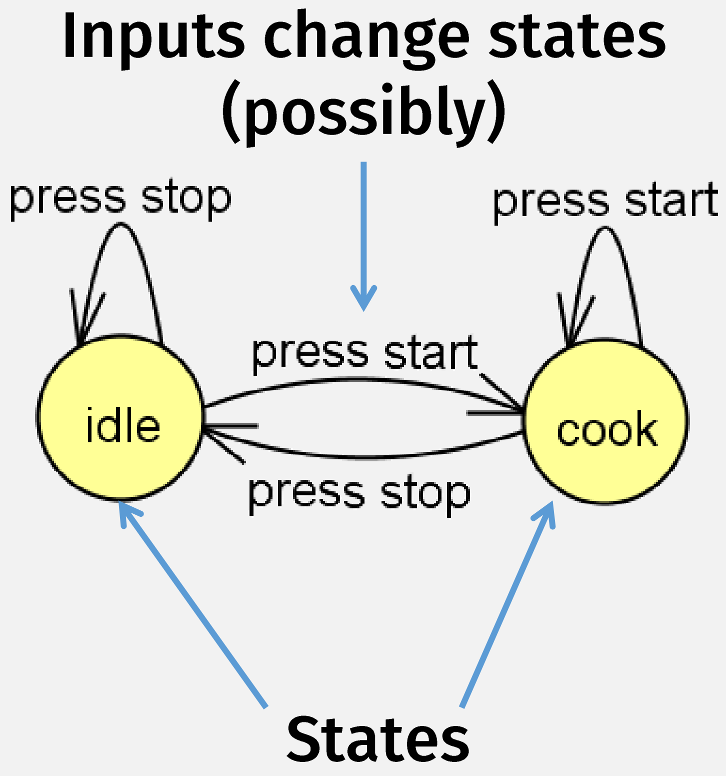
Deterministic Finite Automata (DFA)

State Machine

Finite State Machine (FSM)

Finite Automata: A computational model for …

A Microwave Finite Automata



Finite Automata: Not Just for Microwaves

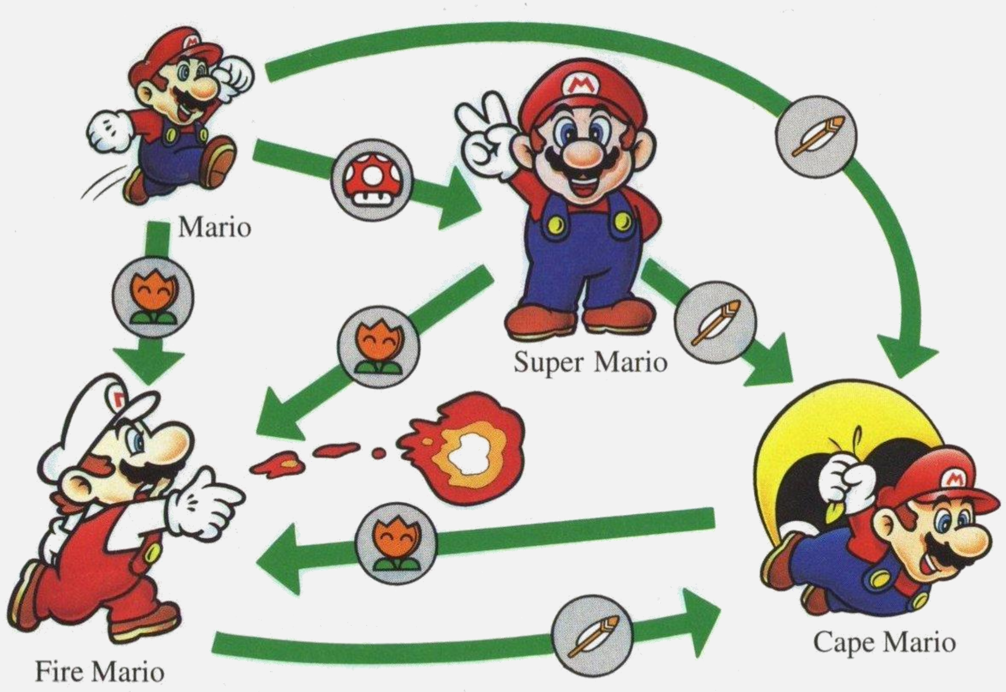
State pattern

From Wikipedia, the free encyclopedia

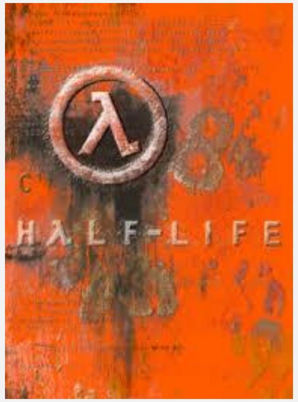
The **state pattern** is a behavioral software design pattern that allows an object to alter its behavior when its internal state changes. This pattern is close to the concept of finite-state machines. The state pattern can be interpreted as a strategy pattern, which is able to switch a strategy through invocations of methods defined in the pattern's interface.

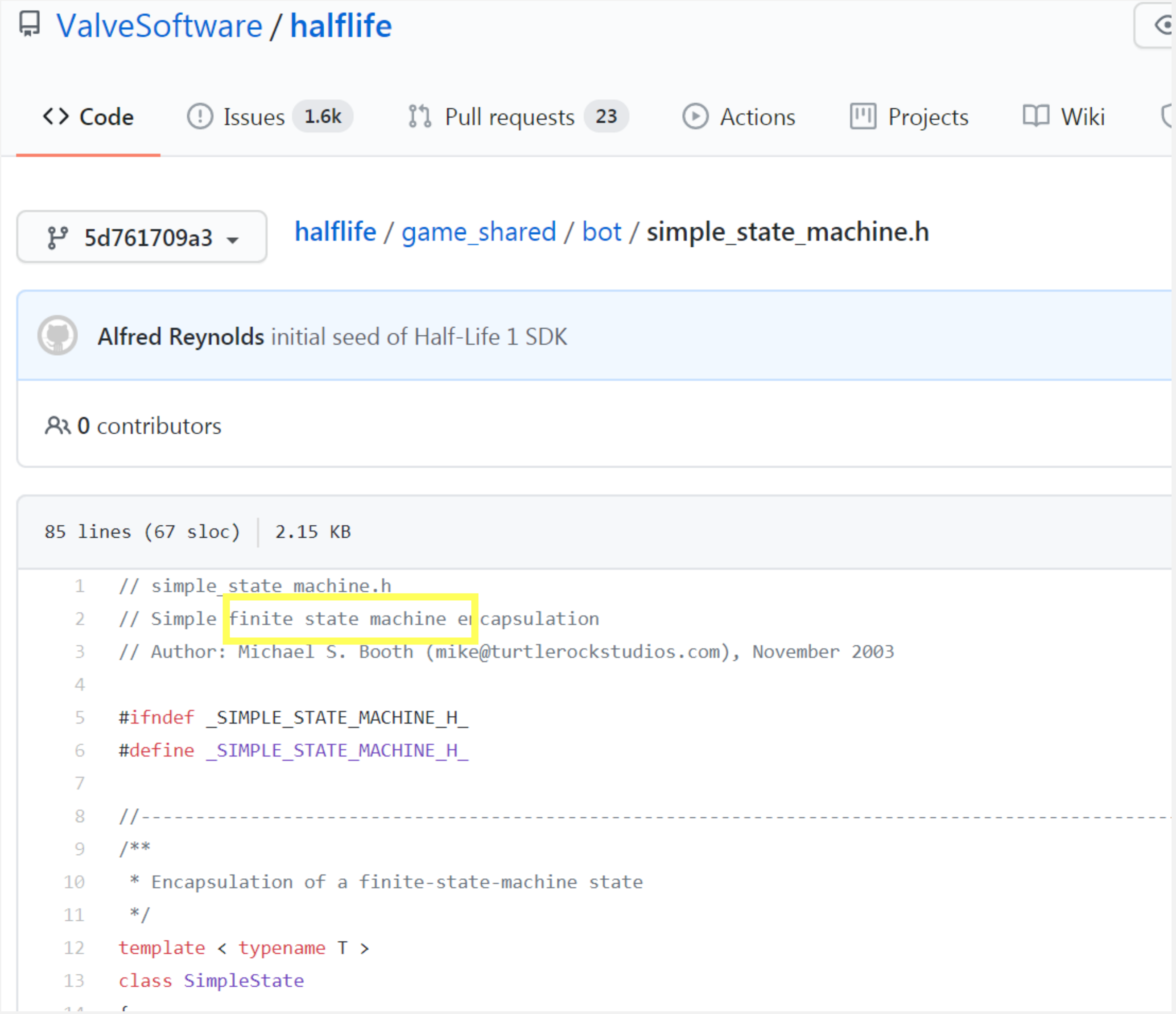
**Finite Automata:
a common
programming pattern**

Video Games Love Finite Automata



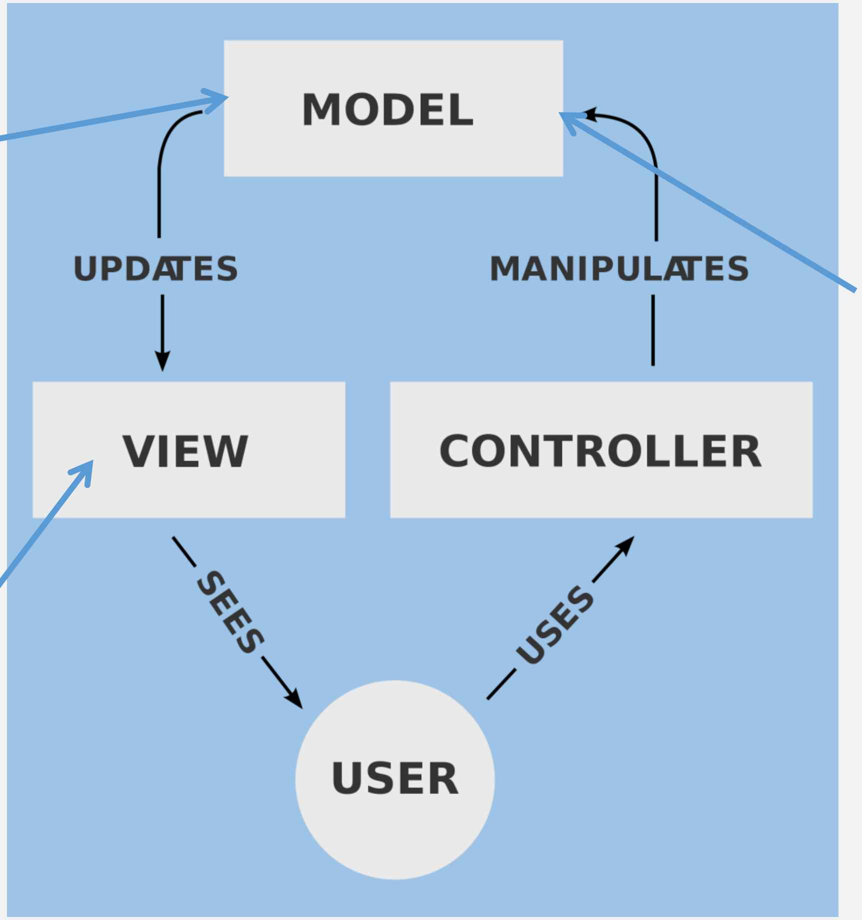
Finite Automata in Video Games





Model-view-controller (MVC) is a FSM

States



Inputs change states

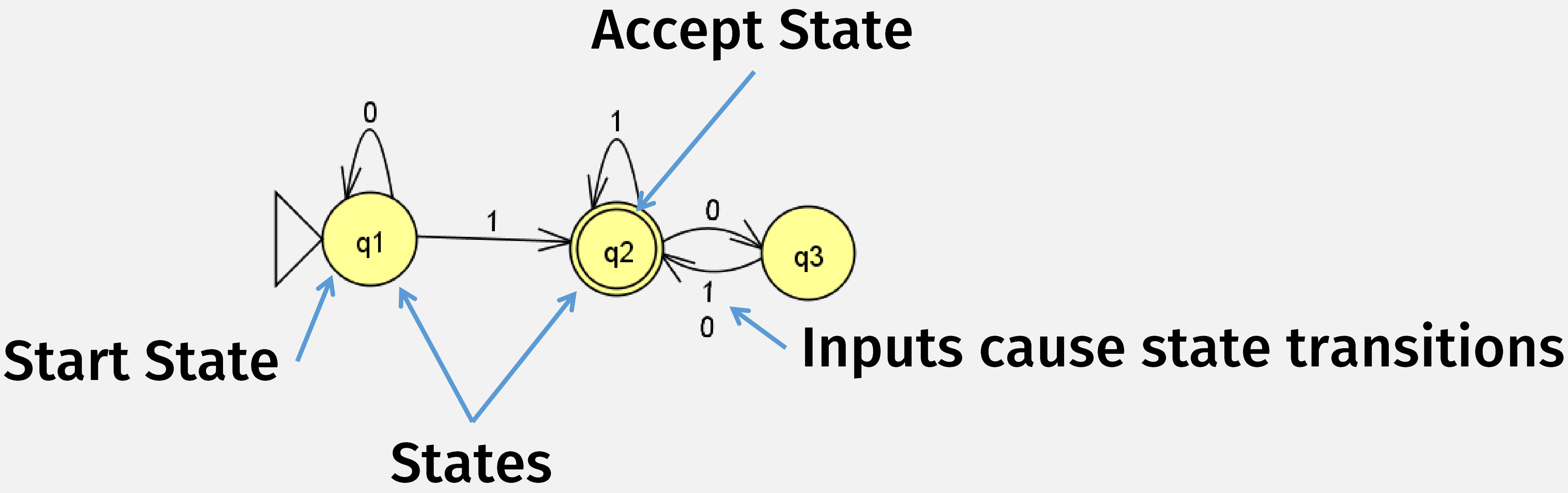
The View Draws states

A Finite Automata is a Computer!

- A very limited computer with finite memory
  - Memory = states

- In this class, we'll formally study automata as:
  - State diagrams
  - Formal mathematical model
  - Code simulations of the mathematical model

Finite Automata state diagram



Finite Automata: The Formal Definition

5 components

DEFINITION 1.5

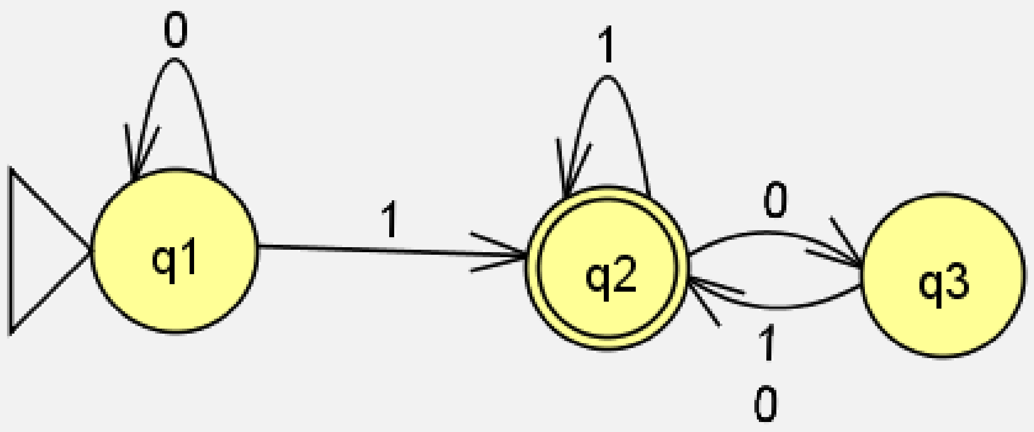
A ***finite automaton*** is a 5-tuple $\boxed{(Q, \Sigma, \delta, q_0, F)}$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta\colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta \colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

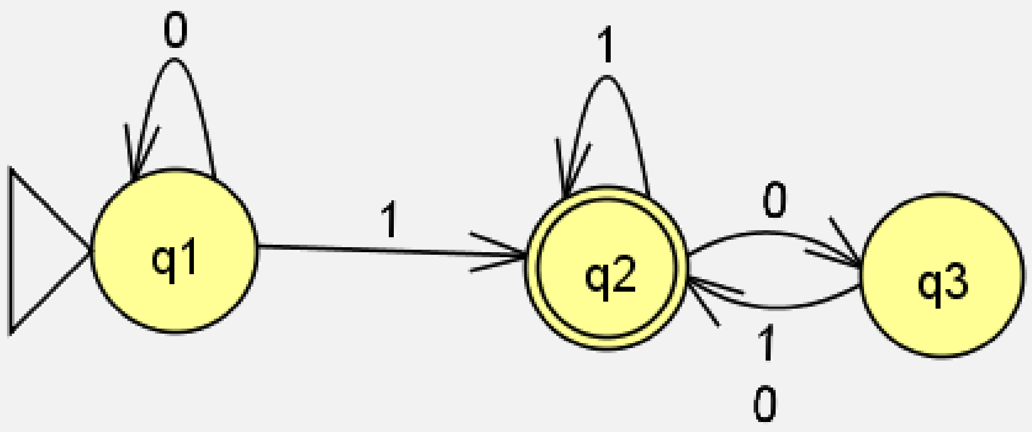


Example: as state diagram

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

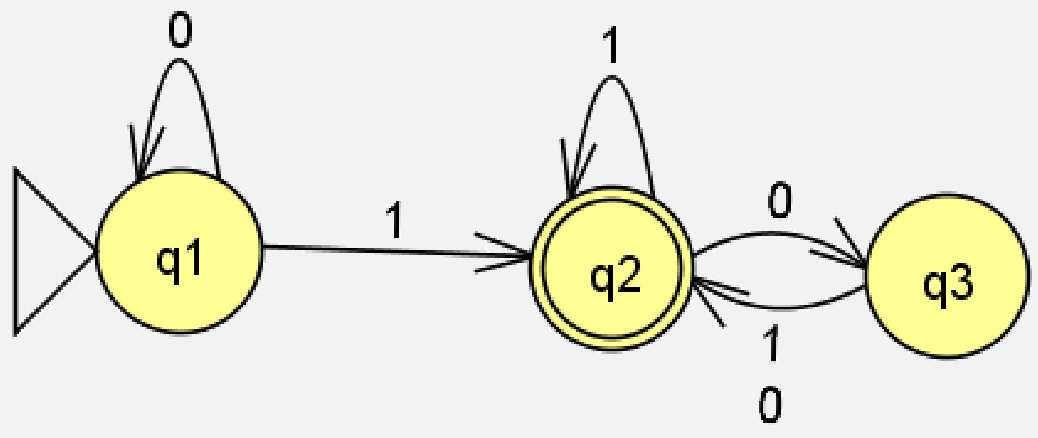
4. q_1 is the start state, and
5. $F = \{q_2\}$.

Braces = Set notation (no duplicates)

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$, ← **Possible inputs**
3. δ is described as

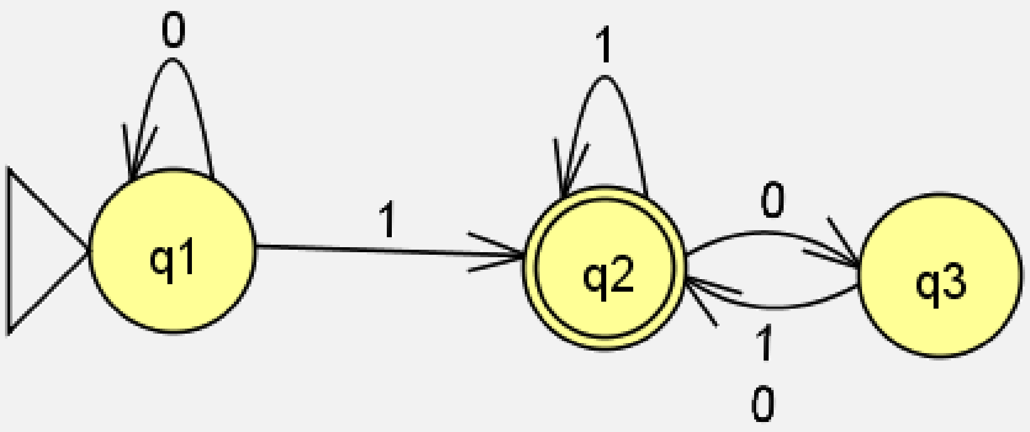
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta\colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

"And this is next input symbol"

"If in this state"

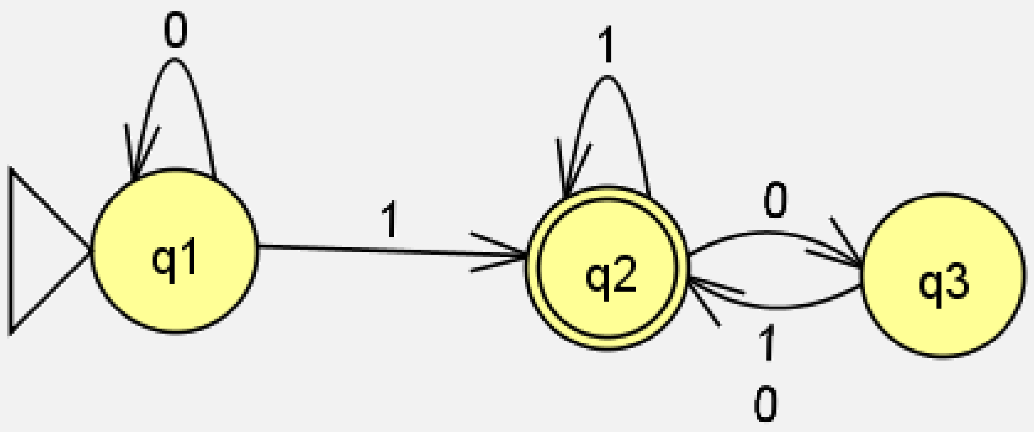
"Then go to this state"

4. q_1 is the start state, and
5. $F = \{q_2\}$.

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta: Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

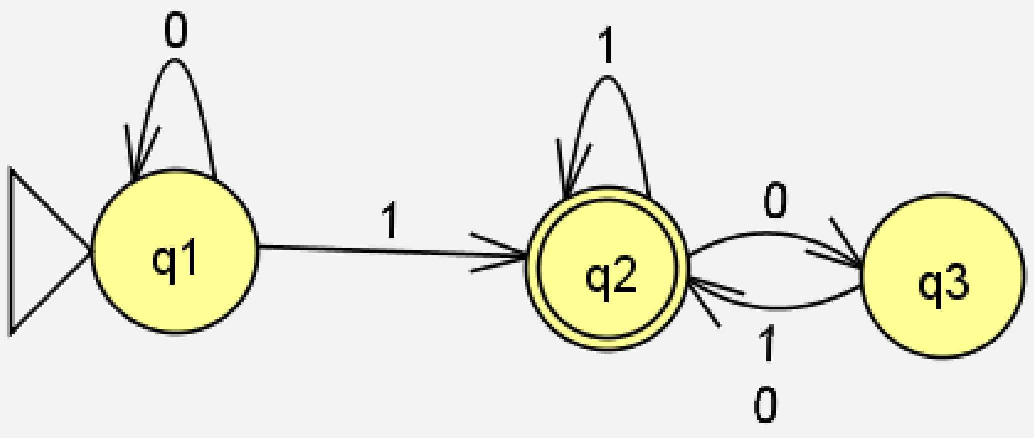
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta: Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

Precise Terminology is Important

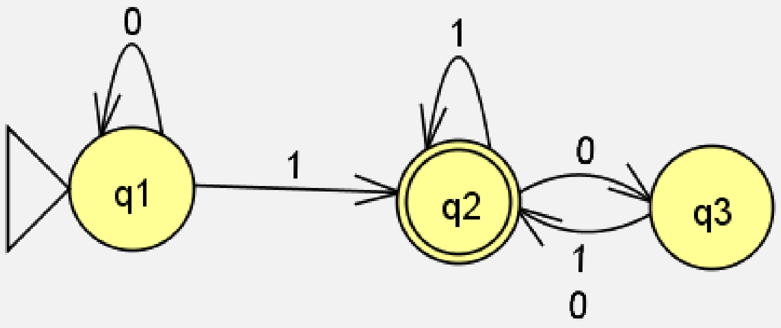
- **<u>Currently</u>**, these terms are all equivalent:
  - Finite State Machine (FSM), State Machine
  - Finite Automaton, Automaton
  - Deterministic Finite Automata (DFA)
- They describe a specific kind of FSM, defined in Definition 1.5

- **<u>Eventually</u>**, we'll learn about many different variations of finite automata:
  - Deterministic Finite Automata (DFA)
  - Non-Deterministic Finite Automata (NFA)
  - Generalized Non-Deterministic Finite Automata (GNFA)

- **<u>Then</u>**, these terms will <u>generally</u> describe the <u>class</u> of machines studied in Ch 1
  - Finite State Machine (FSM), State Machine
  - Finite Automaton, Automaton

- **<u>But</u>** all these machines are related; they are equivalent in "power"

“Running” an FSM “Program” (JFLAP demo)

- **FSM:** 

- **Program:** “`1101`”

The Computation Model

Informally	*Formally (i.e., mathematically)*

- Computer = some finite automata
- Program = input string of chars
- Start in "start state"
- 1 char at a time, follow transition table to change states
- Result =
  - "Accept" if last state is "Accept" state
  - "Reject" otherwise

- $M = (Q, \Sigma, \delta, q_0, F)$
- $w = w_1 w_2 \cdots w_n$
- $r_0 = q_0$
- $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$
- M ***accepts*** w if

sequence of states $r_0, r_1, \ldots, r_n$ in Q exists

with $r_n \in F$

Terminology

- M ***accepts*** w
- M ***recognizes language*** A

  if $A = \{w \mid M \text{ accepts } w\}$

- A language is called a ***regular language*** if some finite automaton recognizes it.

Proving that a language is regular

Kinds of Mathematical Proof

- Proof by construction
  - Construct the mathematical object in question
- Proof by contradiction
- Proof by induction

Proving that a language is regular

- (Usually) requires creating a FSM

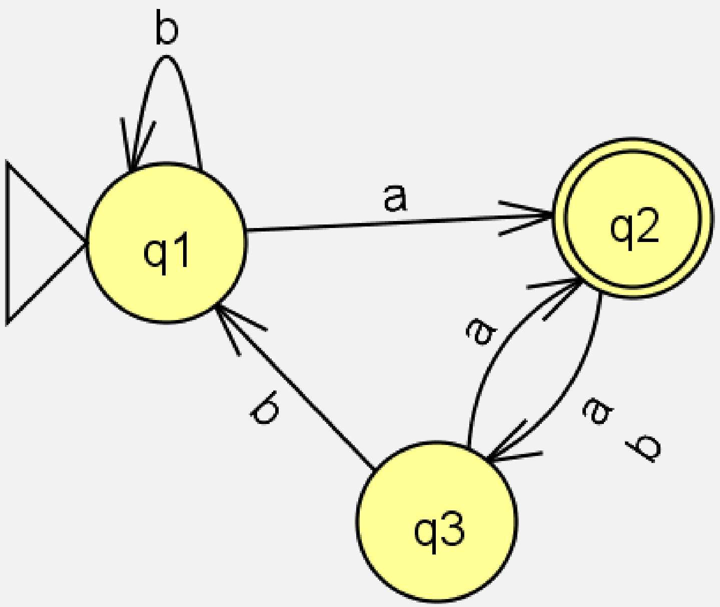
A language is called a ***regular language*** if some finite automaton recognizes it.

Designing Finite Automata

- States = the machine's **memory**!
  - Finite amount of memory: must be allocated in advance
  - Think about what information must be remembered.
- Example: machine accepts strings with even number of 0s
  - Two states: 1) seen even number of 0s, 2) seen odd number of 0s
- Input may only be read once
- Must decide accept/reject after that

In-class exercise 1

- Come up with a formal description of the following machine:



DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta\colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

In-class exercise 2

- Design machine M that recognizes: {w |w has exactly three 1's}
- Where Σ= {0, 1},
- Remember:

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta\colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

Check-in Quiz 1/27

See Gradescope