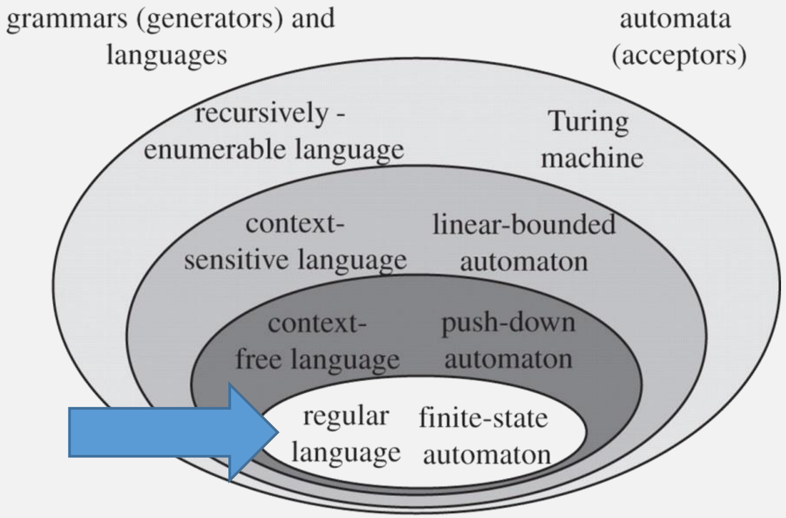
CS420
Regular Languages

Mon Feb 1, 2021



Logistics

- Piazza
  - Also, Discord
- TA: Welcome Anh!
  - Office hours Tues 2:30-4pm, Thurs 3-4:30pm
- Welcome new students
  - Watch old lectures, take old quizzes, do hw0
- HW0 deadline extended to Wed Feb 3 11:59pm EST
- Next HW gets ~1.5 weeks, BUT due in 2 parts:
  - HW1 (out) due: Sun Feb 7 11:59pm EST
  - HW2 (out later) due: Sun Feb 14 11:59pm EST

HW0, So Far: A Makefile

comment

Grader Preinstalled langs:
Python, Java, C, C++, JS, Racket

Targets
(see hw for names)

```
setup:  # install your language here (you can probably leave it blank)

run-hw0-stdio:
	racket hello.rkt # this line must start with a tab

run-hw0-alphabet:
	racket alphabet.rkt

run-hw0-powerset:
	racket powerset.rkt

run-hw0-xml:
	racket xml.rkt
```

Commands to run
(these files better exist)

HW0, So Far

- Read from stdin, write to stdout:
  - Python: `sys.stdin, print`
  - C++: `cin`, `cout`
  - Java: `System.in`, `System.out`
    - `Scanner scanner = new Scanner(System.in).readline` drops newlines!
- Power set of the empty set?
  - The power set of a set S is the set of all possible subsets of S
  - This includes empty set, and S itself!
  - $\mathcal{P}(\{\}) = \{\{\}\}$
- XML parsing:
  - Java: `javax.xml.parsers`
  - Python: `xml.etree.ElementTree: parse and findall`
  - `C++: pugixml`

HW1 Pre-game

- In CS 420 we primarily learn about abstract mathematical objects

- But we may use code as a way to explore these math objects

- So it's important to understand the distinction: math vs code

- E.g., a set is an abstract mathematical object
  - contains other math objects like: strings, nums, characters, and other sets!

- A set's (data) representation in code can take many forms:
  - e.g., a list, an array, a space-separated string (hw0)

Math vs Representation, Examples

Abstract Math Concept	Possible Data Representation
Numbers	
Set	
Tuple (i.e., a small finite set)	
Function, i.e., a set of pairs	
Finite automata	

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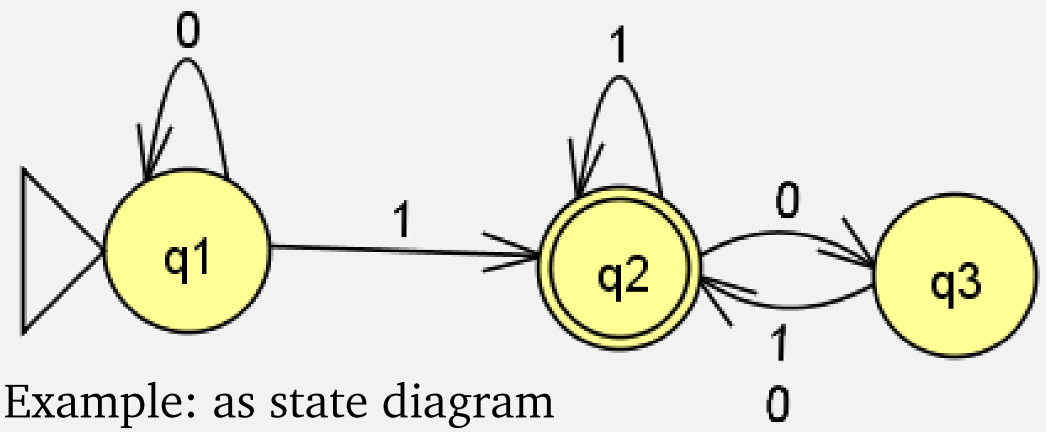
Finite automata	XML str, <your choice here>

Last Time:

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

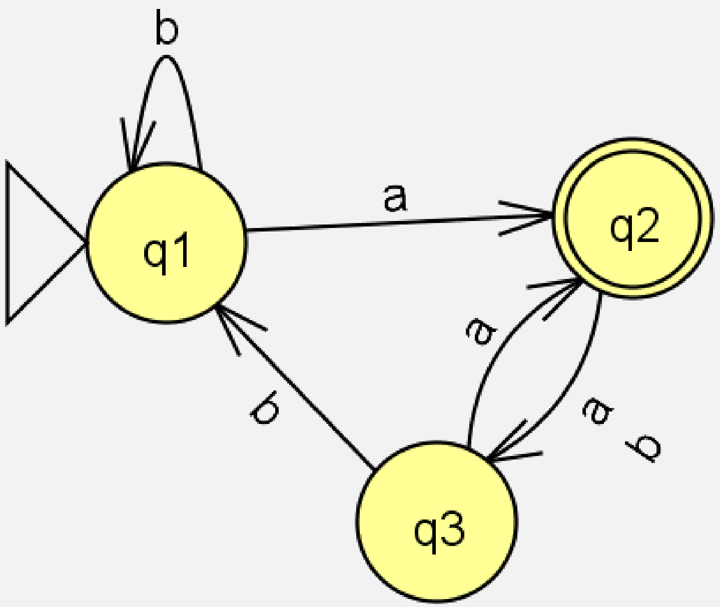
1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

In-class exercise 1

- Come up with a formal description of the following machine:



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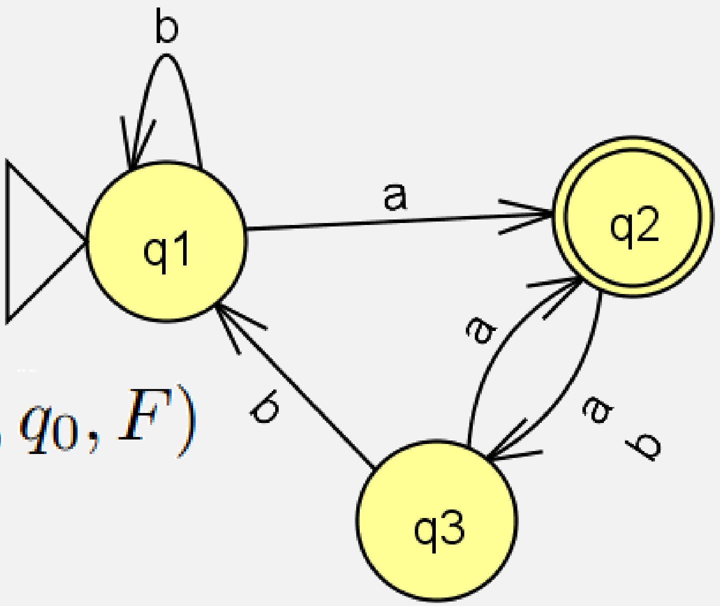
In-class exercise 1: solution



$$M = (Q, \Sigma, \delta, q_0, F)$$

- Q = {q1, q2, q3}
- Σ= {a, b}
- Delta
  - δ(q1,a) = q2
  - δ(q1,b) = q1
  - δ(q2,a) = q3
  - δ(q2,b) = q3
  - δ(q3,a) = q2
  - δ(q3,b) = q1
- q_0 = q1
- F = {q2}

Last Time: Computation, Formally

- A finite automata $M = (Q, \Sigma, \delta, q_0, F)$ is a computer
- We "run" on M an input string $w = w_1 w_2 \cdots w_n$, e.g. "1101"
- M **accepts** w if there is sequence of states $r_0, \ldots, r_n$ in Q where:
  - $r_0 = q_0$ **(start in "start" state)**
  - $\delta(r_i, w_{i+1}) = r_{i+1}, \text{ for } i = 0, \ldots, n-1$ **("next" states follow transition table)**
  - $r_n \in F$ **(last state is an "accept" state)**

A ***language*** is a set of strings.

Terminology

- M ***accepts*** w
- M ***recognizes language*** A

$$\text{if } A = \{w \mid M \text{ accepts } w\}$$

"the set of all …" "such that …"

A ***language*** is a set of strings.

Terminology

- M ***accepts*** w
- M ***recognizes language*** A

  if $A = \{w | M \text{ accepts } w\}$
- A language is called a ***regular language*** if some finite automaton recognizes it.

A *language* is a set of strings.

M ***recognizes language*** A if $A = \{w | M \text{ accepts } w\}$

A language, regular or not?

- <u>If given</u>: Finite Automata M
  - <u>We know</u>: the language recognized by M is a regular language

- <u>If given</u>: some Language A
  - Is A is a regular language?
    - Not necessarily
  - How do we determine, i.e., *prove*, that A is a regular language?

A language is called a ***regular language*** if some finite automaton recognizes it.

Kinds of Mathematical Proof

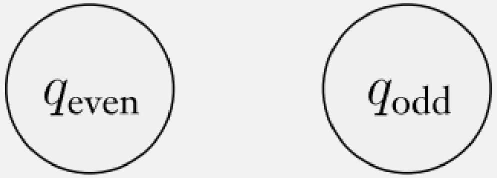
- Proof by construction
  - Construct the mathematical object in question
- Proof by contradiction
- Proof by induction

Designing Finite Automata: Tips

- Input may only be read once
- Must decide accept/reject after that
- States = the machine's **memory**!
  - Finite amount of memory: must be allocated in advance
  - Think about what information must be remembered.
- (For DFAs) Every state/symbol pair must have a transition
- Example: machine accepts strings with odd number of 0s

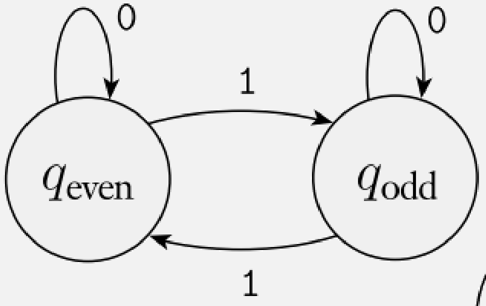
Design a DFA: accepts strs with odd # 0s

- <u>States</u>:
  - 2 states:
    - seen even 0s so far
    - seen odds 0s so far

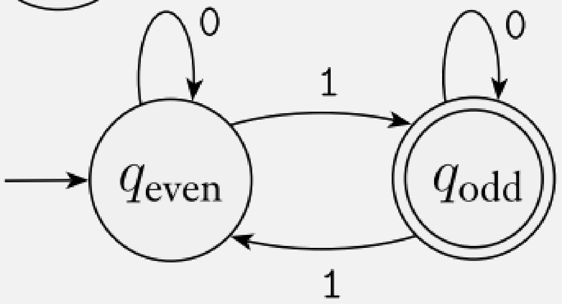


- <u>Alphabet</u>: 0 and 1

- <u>Transitions</u>:



- <u>Start</u> / <u>Accept</u> states?



In-class exercise 2

- Prove that this language is a regular language:
  - {w | w has exactly three 1's}
  - i.e., design a finite automata that recognizes it!

- Where $\Sigma = \{0, 1\}$,

- Remember:

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Check-in Quiz 2/1

See Gradescope