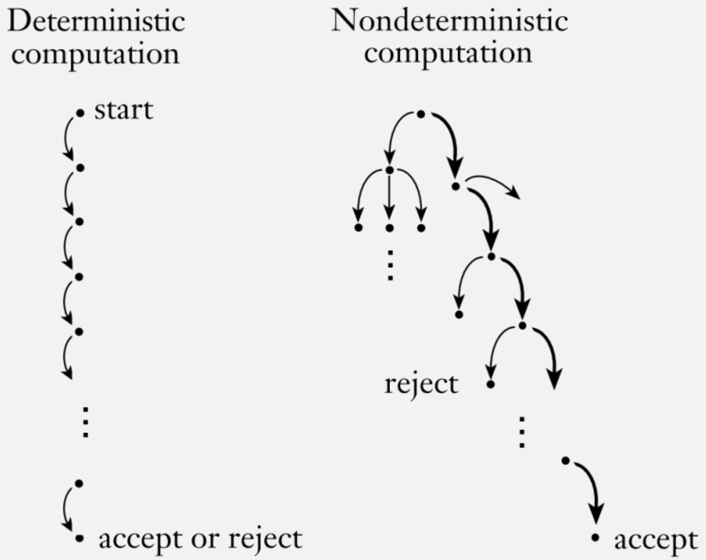
Nondeterminism

Monday Feb 8, 2021



Logistics

- HW 0, HW 1 done
- HW 2 due Sunday 2/14 11:59pm EST
- No class next Monday 2/15
- (Some) HW 0 solutions posted
- Questions?

A Brief Intro to XML

- What is it?
  - It's a widely-used "data interchange format"
    - I.e., A standard, language-agnostic way for programs to send/recv data
  - (JSON is another popular interchange format)

- Example, when querying web apis:
  - `https://api.etrade.com/v1/market/quote/GOOG`


```xml
<?xml version="1.0" encoding="UTF-8"?>
<QuoteResponse>
   <QuoteData>
      <dateTime>15:17:00 EDT 06-20-2018</dateTime>
      <dateTimeUTC>1529522220</dateTimeUTC>
      <quoteStatus>DELAYED</quoteStatus>
      <ahFlag>false</ahFlag>
      <hasMiniOptions>false</hasMiniOptions>
      <All>
         <adjustedFlag>false</adjustedFlag>
         <annualDividend>0.0</annualDividend>
         <ask>1175.79</ask>
         <askExchange />
         <askSize>100</askSize>
         <askTime>15:17:00 EDT 06-20-2018</askTime>
         <bid>1175.29</bid>
         <bidExchange />
         <bidSize>100</bidSize>
         <bidTime>15:17:00 EDT 06-20-2018</bidTime>
```

XML in this class: 2 purposes

1. Grader uses it to send/get state machines to/from HW
   - E.g.

```
<automaton>
  <!--The list of states.-->
  <state id="0" name="q1"><initial/></state>
  <state id="1" name="q2"><final/></state>
  <state id="2" name="q3"></state>

  <!--The list of transitions.-->
  <transition>
    <from>0</from>
    <to>0</to>
    <read>0</read>
  </transition>

  <transition>
    <from>1</from>
```

Open tags may contain attributes

attribute name

attribute value

Elements nest, i.e., they may contain other elements

element = open/close tag + everything in between

Open tag

Close tag

XML in this class: 2 purposes

A ***language*** is a set of strings.

M ***recognizes language*** A if $A = \{w |\ M \text{ accepts } w\}$

2. Running example of a “language”, to compare/contrast computation models
   - E.g.,

```
<automaton>
  <!--The list of states.-->
  <state id="0" name="q1"><initial/></state>
  <state id="1" name="q2"><final/></state>
  <state id="2" name="q3"></state>

  <!--The list of transitions.-->
  <transition>
    <from>0</from>
    <to>0</to>
    <read>0</read>
  </transition>

  <transition>
    <from>1</from>
```

“Language” of all possible open tag strings is regular

“Language” of all XML strings is <u>not</u> regular, because a DFA cannot do open/close tag matching

Last time: "Closed" Operations

> A set is **closed** under an operation if applying the operation to members of the set returns an element still in the set

- E.g., Natural numbers = {0, 1, 2, …}
  - closed under addition,
  - not closed under subtraction

Last time: Union is Closed for Reg. Langs

THEOREM 1.25

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Proof (implement this algorithm for HW2)

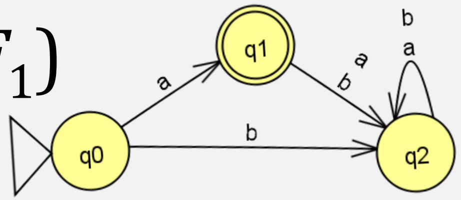
- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,
- Construct a new machine $M = (Q, \Sigma, \delta, q_0, F)$ using M_1 and M_2

M simulates running its input on both M_1 and M_2 in "parallel"; accept if either accepts

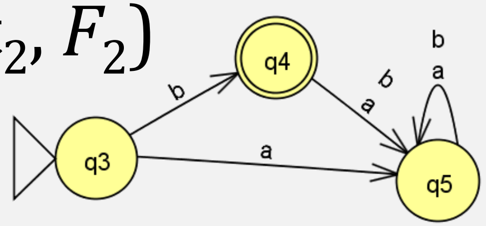
DFA Union Example

- $A_1 = \{\text{“a”}\}, \quad A_2 = \{\text{“b”}\}, \quad A_1 \cup A_2 = \{\text{“a”, ”b”}\}$

- $M_1 = (Q_1, \Sigma, \delta_1, \text{start}_1, F_1)$



$M_2 = (Q_2, \Sigma, \delta_2, \text{start}_2, F_2)$



- M recognizing $\{\text{“a”, “b”}\} = (Q, \Sigma, \delta, \text{start}, F)$
  - $Q = Q_1 \times Q_2 = \{(\text{q0}, \text{q3}), (\text{q0}, \text{q4}), (\text{q0}, \text{q5}), \ldots\}$
  - $\Sigma = \{\text{a}, \text{b}\}$
  - $\delta((\text{q0}, \text{q3}), \text{a}) = (\delta_1(\text{q0}), \delta_2(\text{q3})) = (\text{q1}, \text{q5})$
    - …
  - $\text{start} = (\text{q0}, \text{q3})$
  - $F = \{(\text{q1}, \text{q3}), (\text{q1}, \text{q4}), (\text{q1}, \text{q5}), (\text{q4}, \text{q0}), \ldots\}$

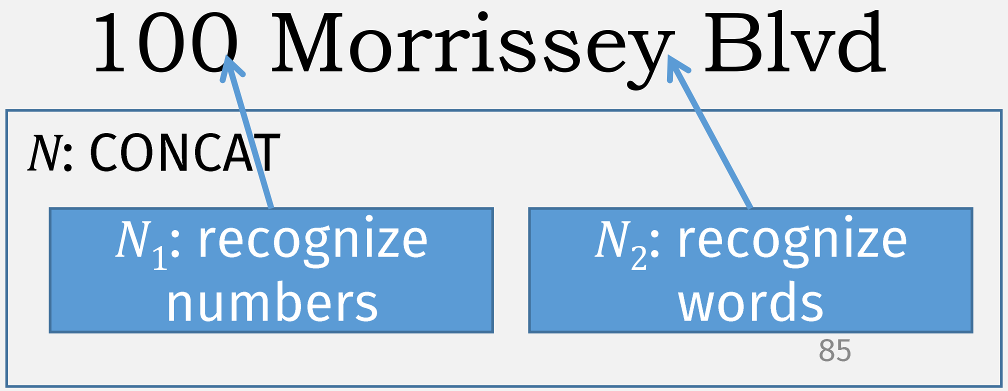
<u>Last time</u>: Is Concatenation Closed?

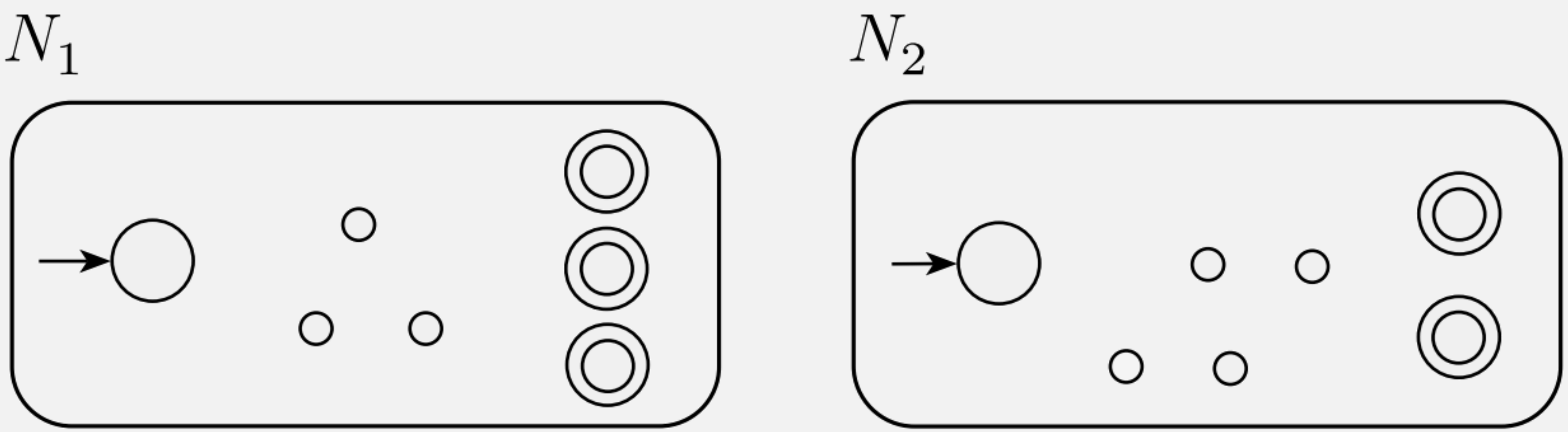
THEOREM 1.26

The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

- <u>Proof</u>: Construct a <u>new</u> machine? (like union)
- How does N know when to switch from N_1 to N_2?
  - Can only read input once





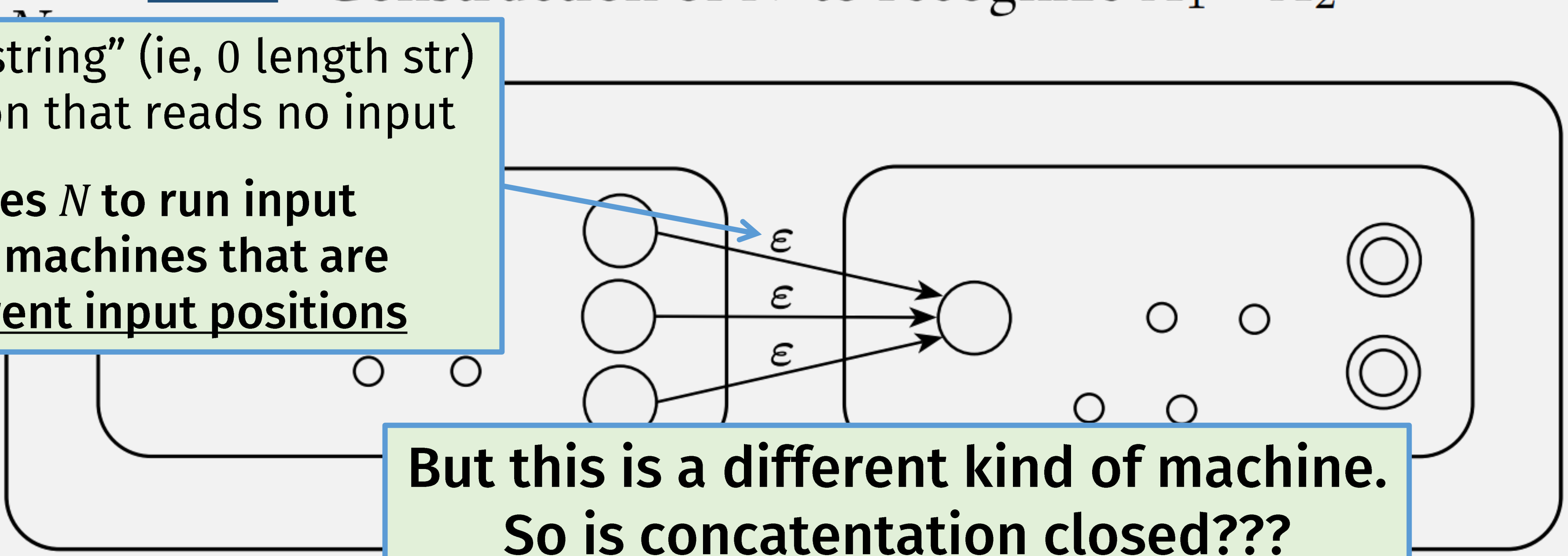
Let N_1 recognize A_1, and N_2 recognize A_2.

N must <u>simultaneously</u>:
- Keep checking with N_1 **and**
- Move to N_2 to check 2nd part

<u>Want</u>: Construction of N to recognize $A_1 \circ A_2$

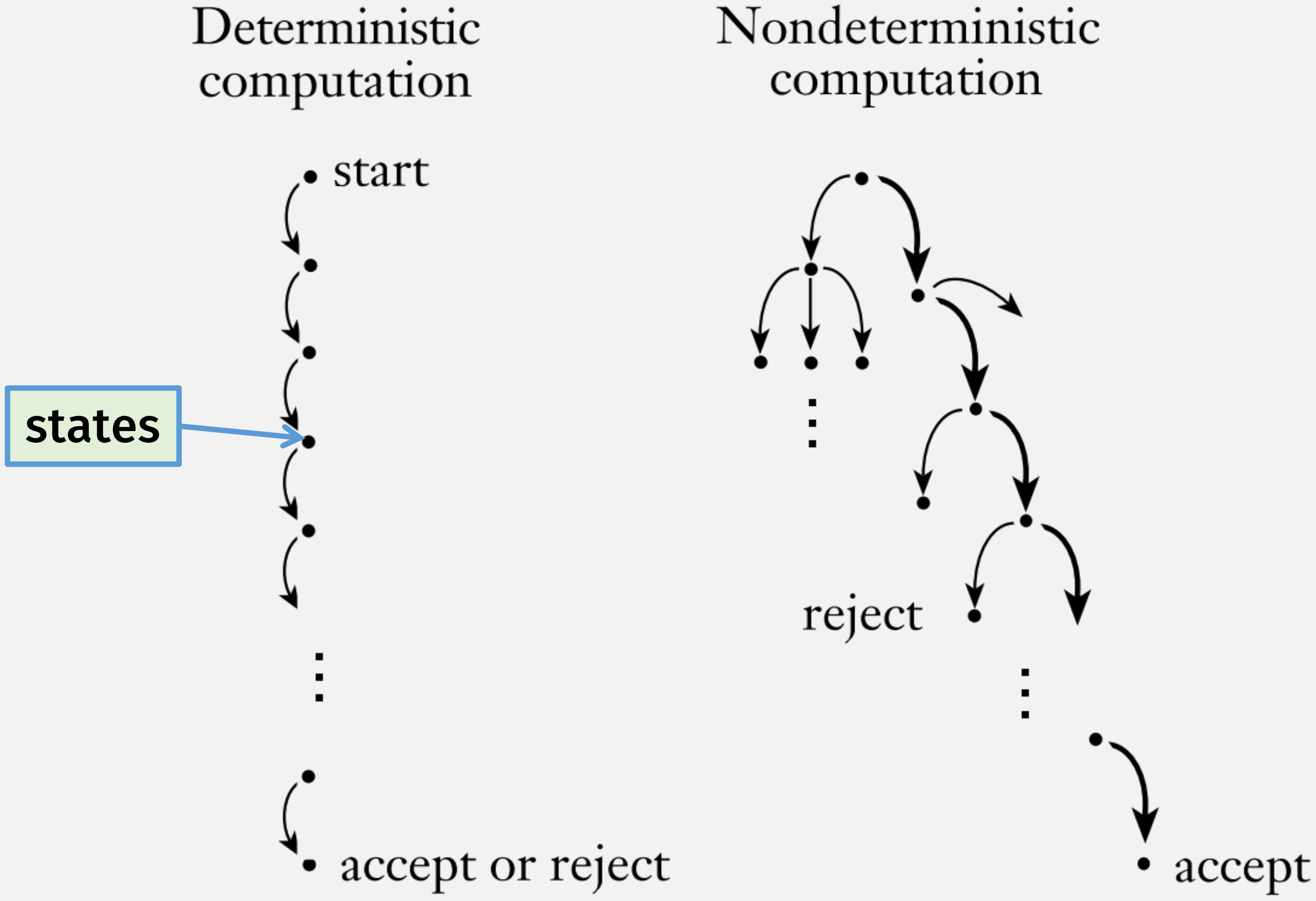
ε = "empty string" (ie, 0 length str)
= transition that reads no input

Enables N to run input on two machines that are <u>at different input positions</u>

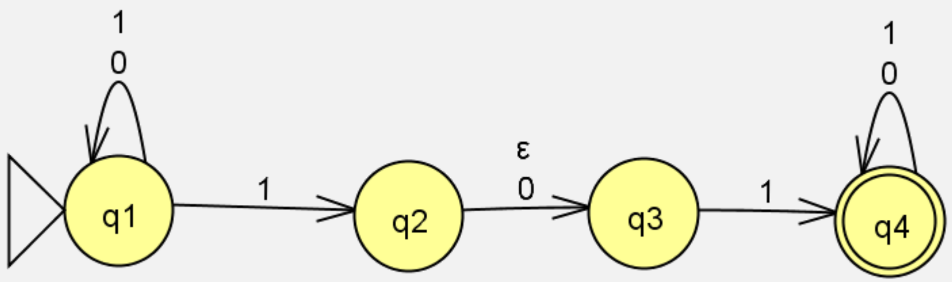


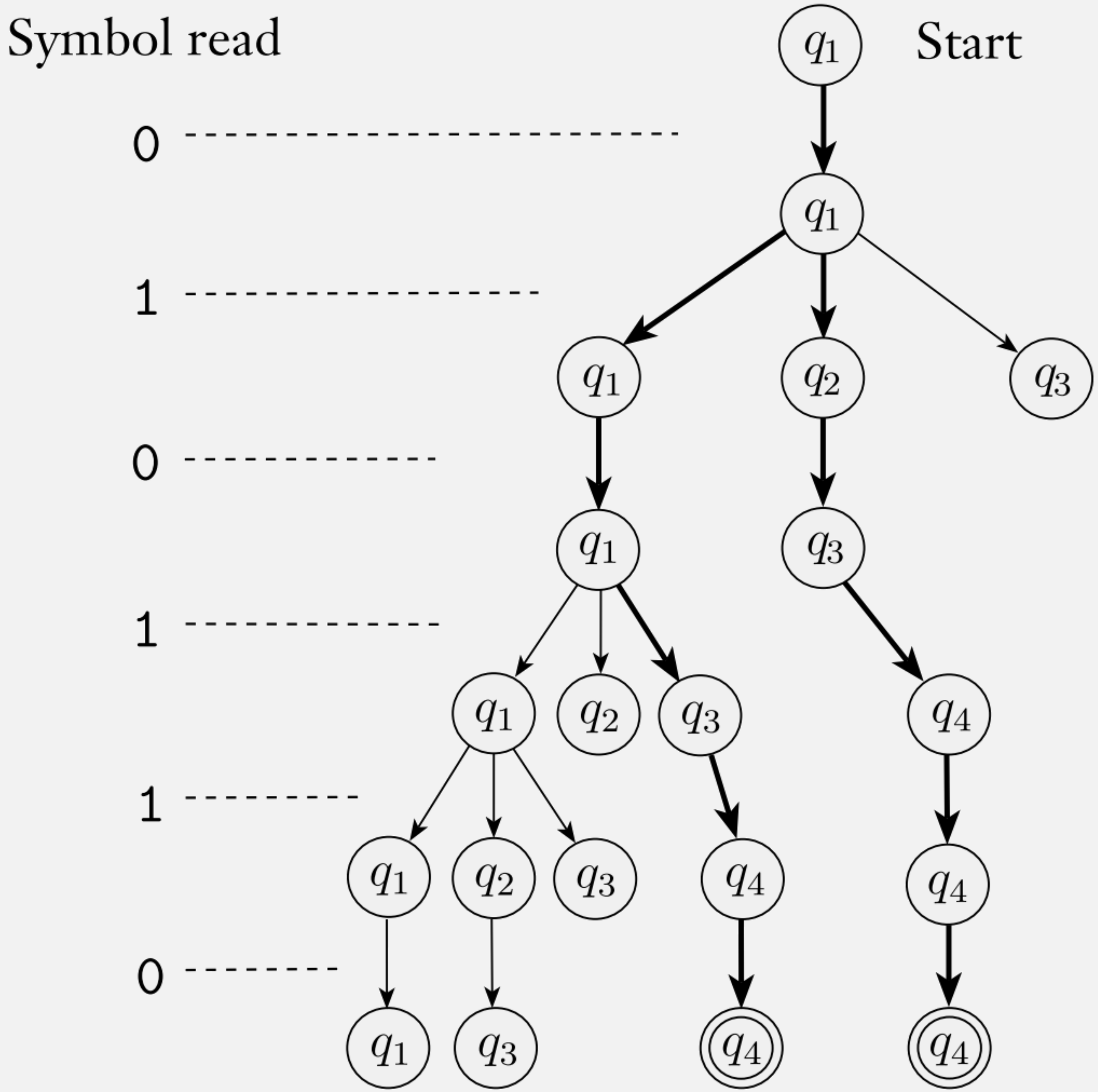
But this is a different kind of machine. So is concatentation closed???

Nondeterminism



Example Fig 1.27 (JFLAP demo): 010110




Symbol read
Start
0
1
0
1
1
0
q_1
q_2
q_3
q_4

Nondeterministic machine can be in multiple states at once

DEFINITION 1.37

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta \colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

Power set

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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5. $F \subseteq Q$ is the ***set of accept states***.

Power Sets

- A power set is the set of all subsets of a set
- Example: `S = {a,b,c}`
- Power set of S =
  - `{{},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}`

Formal Definition of "Computation"

- DFA (from before): Let $w = w_1 w_2 \cdots w_n$

M ***accepts*** w if a sequence of states $r_0, r_1, \ldots, r_n$ in Q exists with three conditions:

1. $r_0 = q_0$,
2. $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$, and
3. $r_n \in F$.

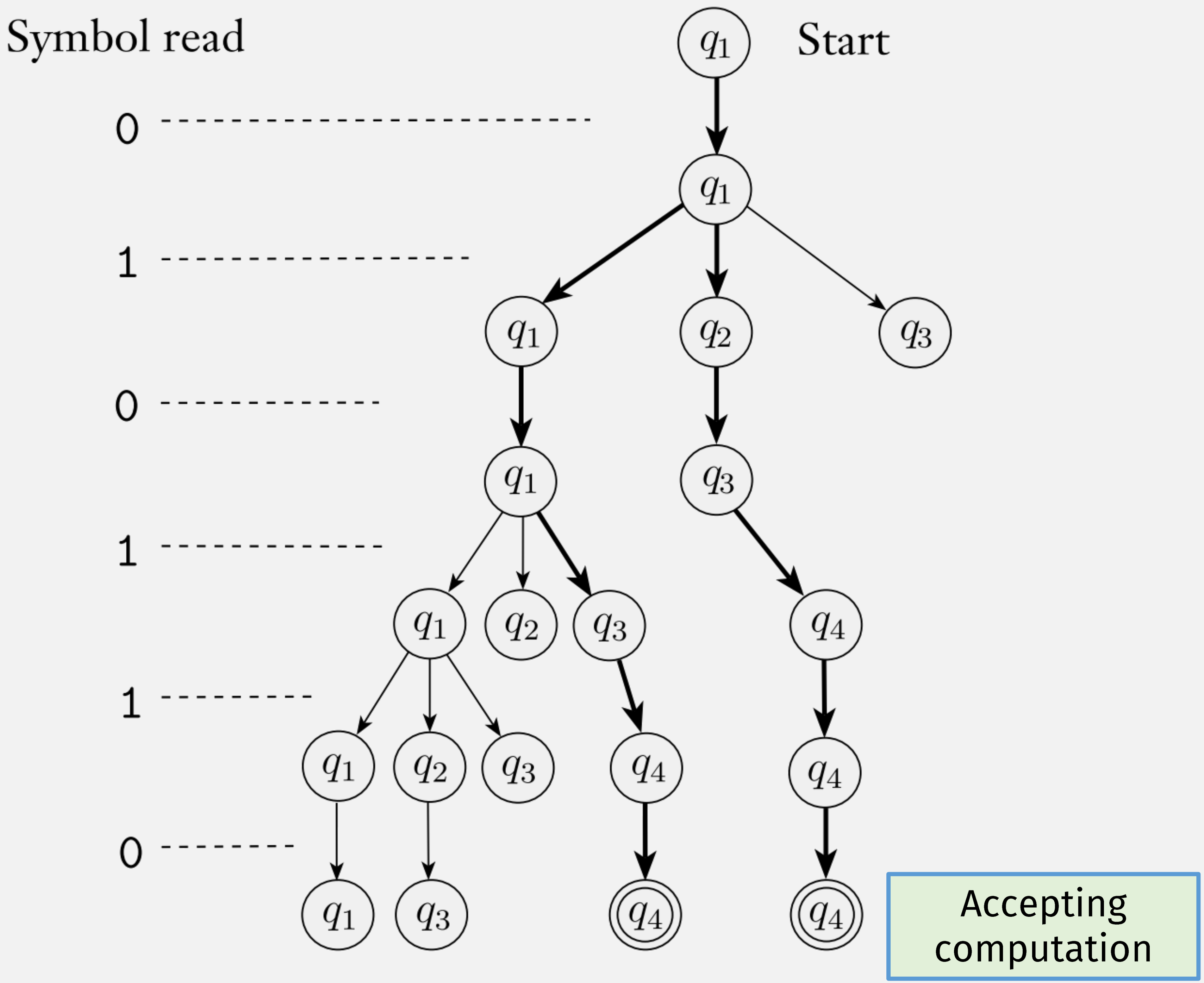
- NFA: Let $w = y_1 y_2 \cdots y_m$

N ***accepts*** w if a sequence of states $r_0, r_1, \ldots, r_m$ exists in Q with three conditions:

1. $r_0 = q_0$,
2. $r_{i+1} \in \delta(r_i, y_{i+1})$, for $i = 0, \ldots, m-1$, and
3. $r_m \in F$.

This is now a set

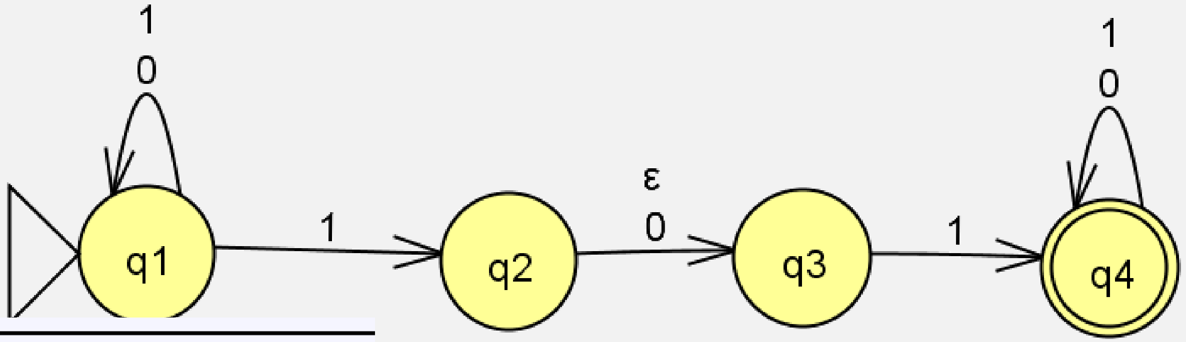
Nondeterministic computation requires **only one path to accept state** in the computation tree



Symbol read
Start
0
1
0
1
1
0
q_1
q_2
q_3
q_4
Accepting computation

In-class exercise

- Come up with a formal description of the following NFA:



DEFINITION 1.37

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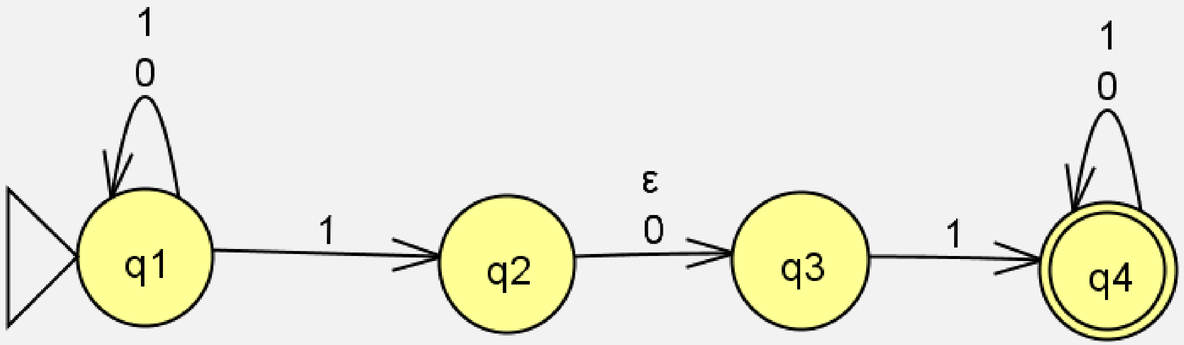
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

The formal description of N_1 is $(Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3, q_4\}$,
2. $\Sigma = \{0,1\}$,
3. δ is given as
4. q_1 is the start state, and
5. $F = \{q_4\}$.



So is Concatenation Closed for Reg Langs?

- Concatenation of DFAs produces an NFA
- But: A language is called a ***regular language*** if some DFA recognizes it.
- To show that concatenation is closed for regular languages, we must prove that NFAs *also* recognize regular languages.
- Specifically, we must prove:
  - NFAs ⇔ regular languages

How to Prove a Theorem: $X \Leftrightarrow Y$

- $X \Leftrightarrow Y$ = "X if and only if Y" = X iff Y = X <=> Y
- Proof at minimum has 2 parts:

1. **=>** if X, then Y
   - i.e., assume X, then use it to prove Y
   - "forward" direction
2. **<=** if Y, then X
   - i.e., assume Y, then use it to prove X
   - "reverse" direction

Proving NFAs recognize regular langs

- *Theorem*:
  - A language A is regular **if and only if** some NFA N recognizes it.

- Must prove:
  - **=>** If A is regular, then some NFA N recognizes it
    - Easy
    - We know: if A is regular, then a **DFA** recognizes it.
    - Easy to convert DFA to an NFA! (how?)
  - **<=** If an NFA *N* recognizes A, then A is regular.
    - Hard
    - Idea: Convert NFA to DFA

Need a way to convert NFA -> DFA

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
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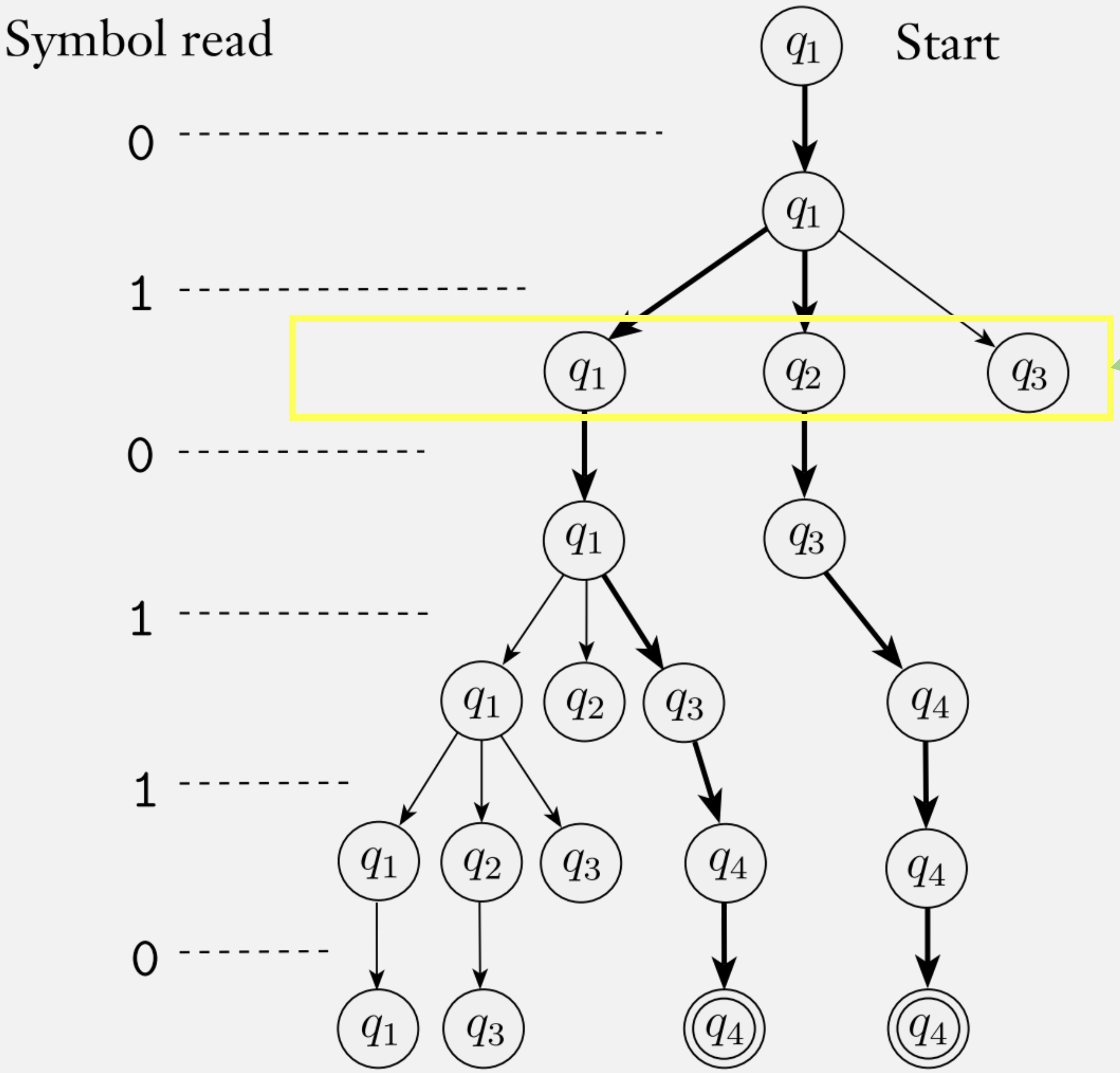
Proof idea:
Each "state" of the DFA must be a set of states in the NFA



In a DFA, all these states at each step must be only **one** state

So design a state in the converted DFA to be a **set of NFA states**!

Example:

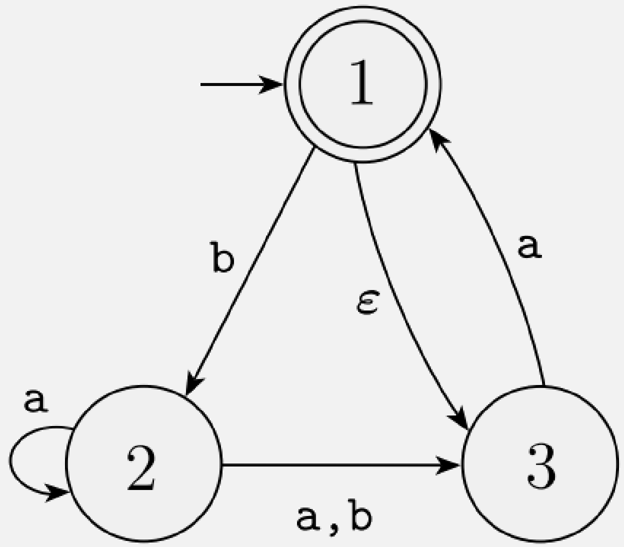


FIGURE 1.42
The NFA N_4

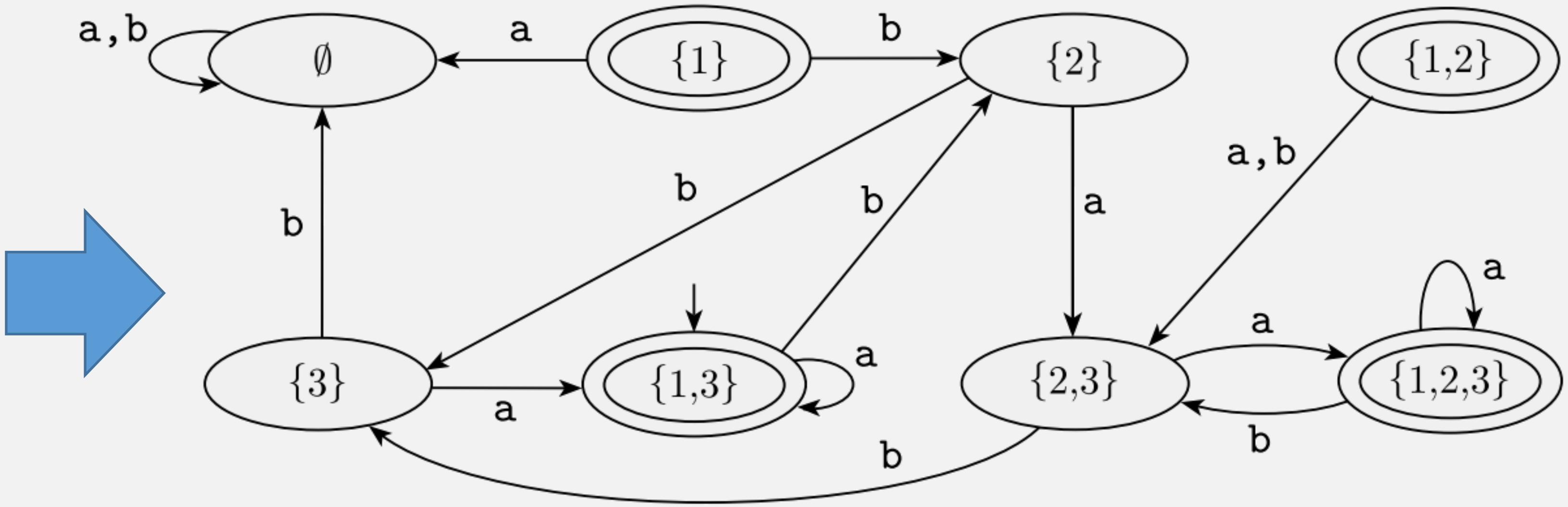


FIGURE 1.43
A DFA D that is equivalent to the NFA N_4

Next time: Convert NFA -> DFA, Formally

- Let NFA N = $(Q, \Sigma, \delta, q_0, F)$
- Then equivalent DFA M has states Q' = $\mathcal{P}(Q)$ (power set of Q)
- (implement this algorithm for HW3)

Check-in Quiz 2/8

On gradescope

In-Class Survey

See course website