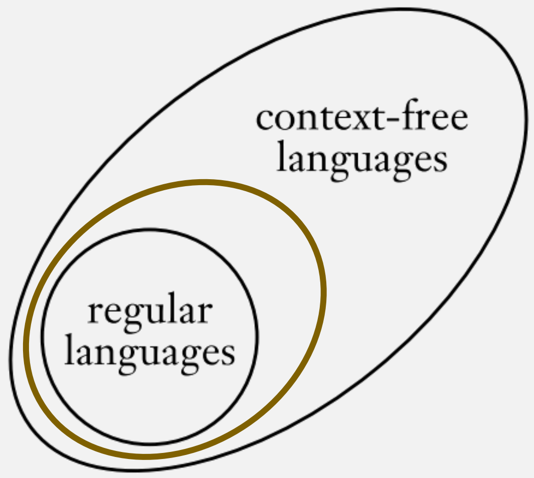
Regular vs Context-Free Languages
(and others?)

Monday, March 20, 2023

UMB CS 420



Announcements

- HW 5 in
  - ~~Due Sunday March 19, 2023 11:59pm EST~~

- HW 6 out
  - Due Sunday March 26, 2023 11:59pm EST

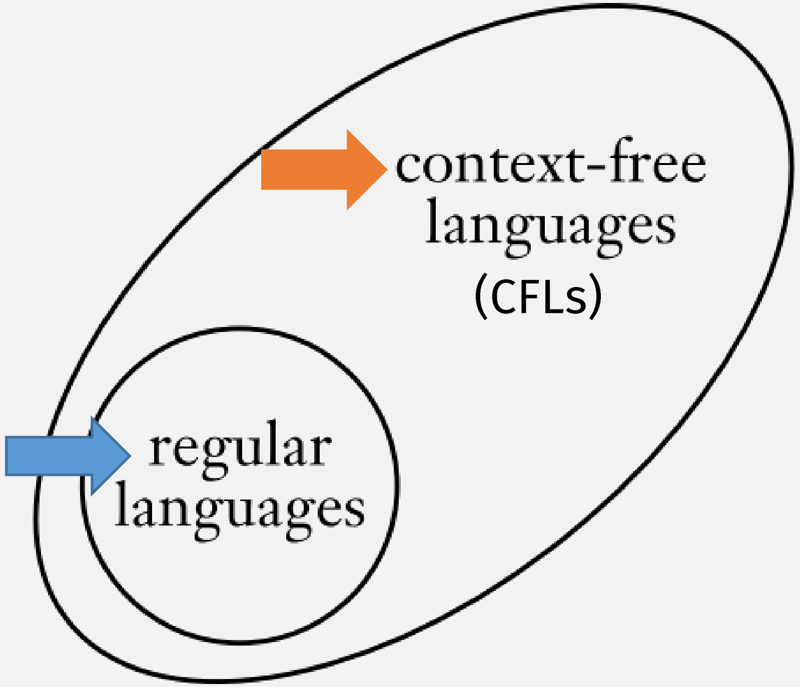
Quiz Preview

- Is every CFL a regular language?

Is This Diagram "Correct"?

(What are the statements implied by this diagram?)

1. Every regular language is a CFL
2. Not every CFL is a regular language



How to Prove This Diagram "Correct"?

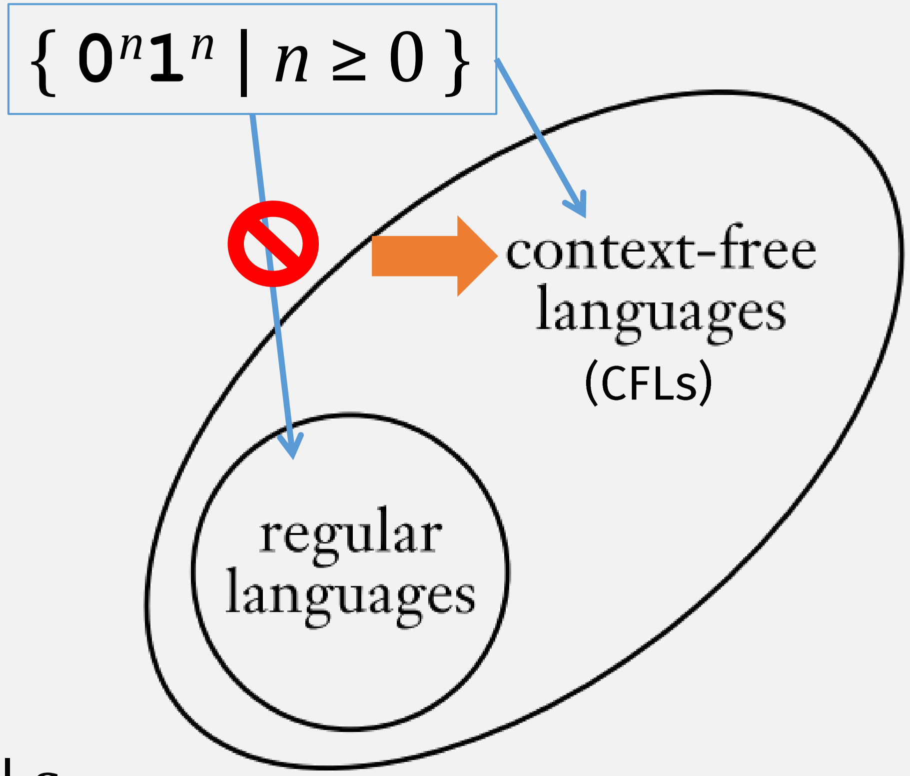
1. Every regular language is a CFL

2. Not every CFL is a regular language

Find a CFL that is not regular

$\{ \mathtt{0}^n\mathtt{1}^n \mid n \geq 0 \}$

- It's a CFL
  - *Proof*: CFG $S \to \mathtt{0}S\mathtt{1} \mid \varepsilon$
- It's not regular
  - *Proof*: by contradiction using the Pumping Lemma



How to Prove This Diagram "Correct"?

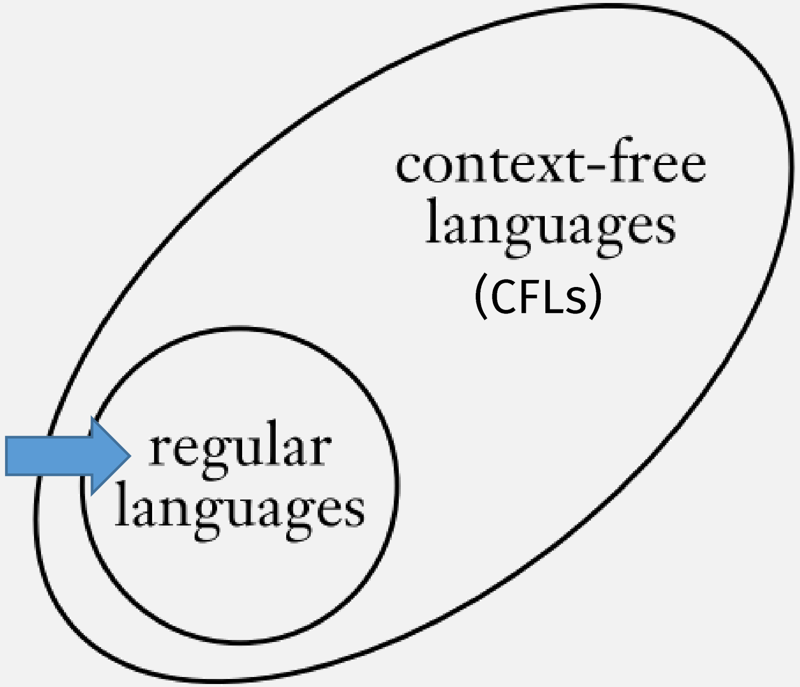
1. Every regular language is a CFL

For any regular language A, show …

… it has a CFG or PDA

☑ 2. Not every CFL is a regular language

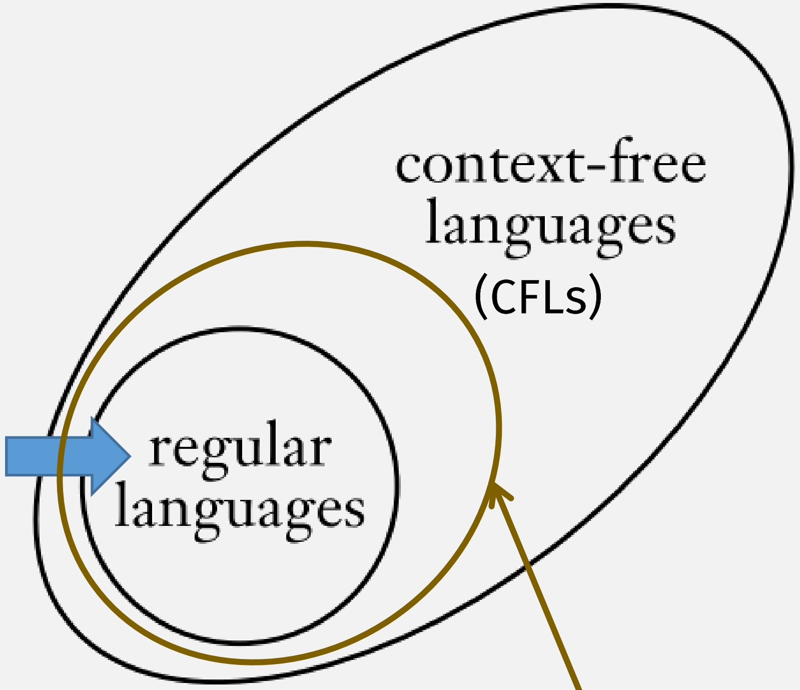
A regular language is represented by a:
- DFA
- NFA
- Regular Expression



Regular Languages are CFLs: 3 Ways to Prove

- DFA → CFG or PDA
- NFA → CFG or PDA
- Regular expression → CFG or PDA

See HW 6!



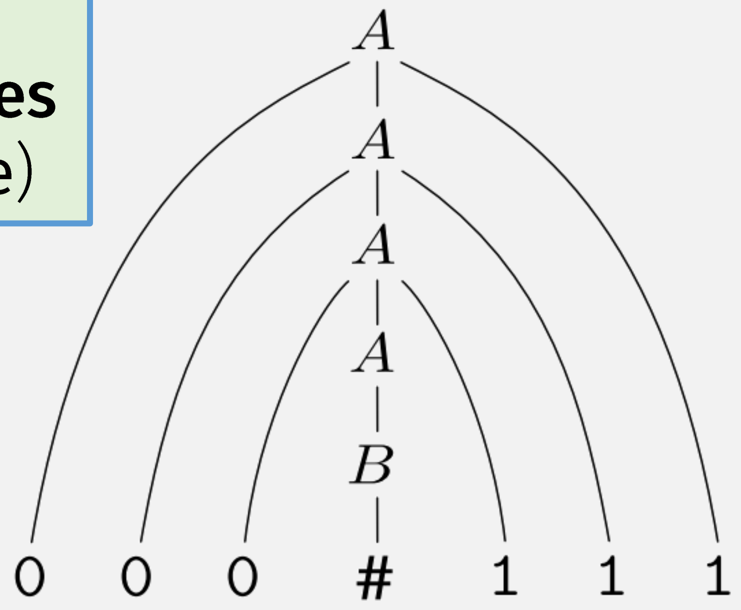
Are there other interesting subsets of CFLs?

Deterministic CFLs and DPDAs

Previously: Generating Strings

Generating strings:
1. Start with **start variable,**
2. Repeatedly *apply* **CFG rules**
to get string (and **parse tree**)

$$A \rightarrow 0A1$$
$$A \rightarrow B$$
$$B \rightarrow \#$$

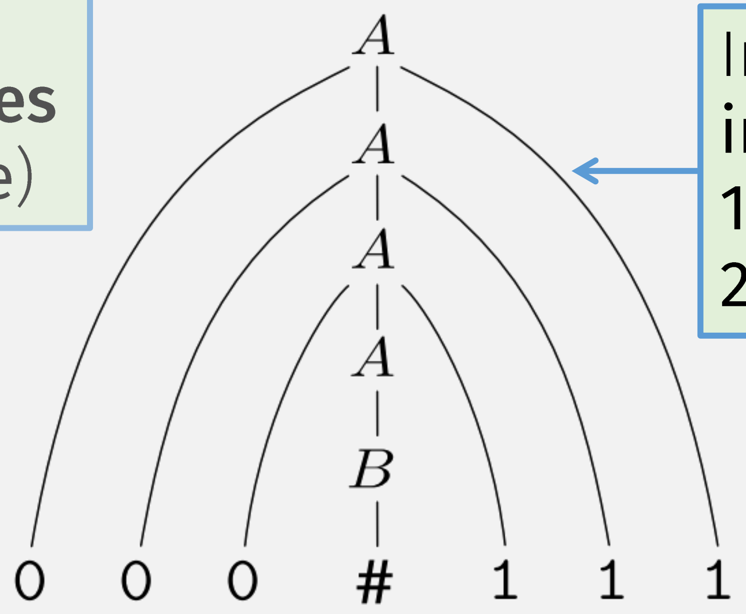


$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\#111$$

Generating vs Parsing

Generating strings:
1. Start with **start variable,**
2. Repeatedly *apply* **CFG rules** to get string (and parse tree)

$$A \to 0A1$$
$$A \to B$$
$$B \to \#$$



In practice, opposite is more interesting:
1. Start with a **string,**
2. Then **parse** it into **parse tree**

$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\#111$$

Generating vs Parsing

- In practice, **parsing** a string more important than **generating** one
  - E.g., a **compiler** (first) parses source code into a parse tree
  - (Actually, *any* program with string inputs must first parse it)

Previously: Example **CFG→PDA**

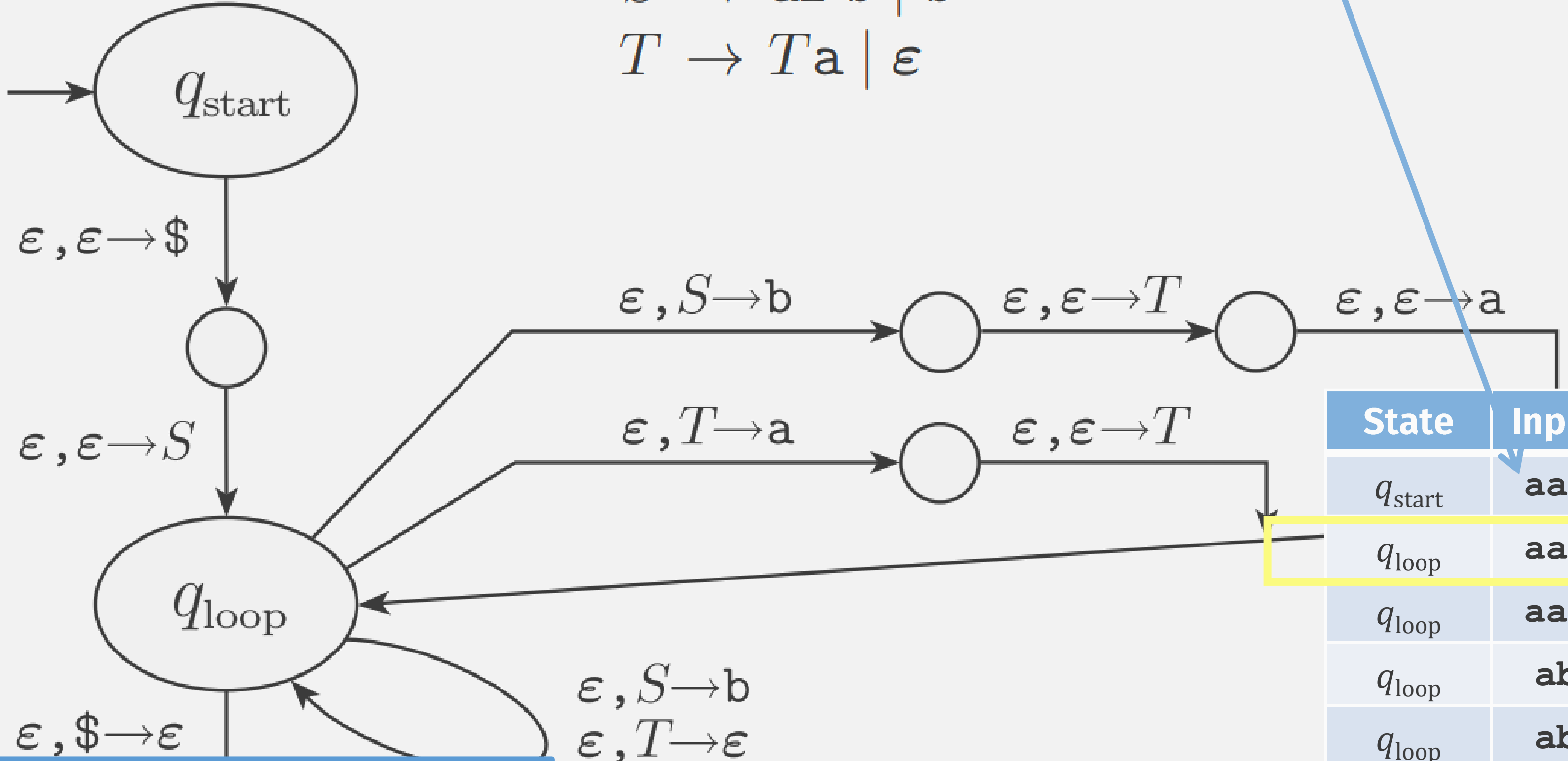
Example Derivation using CFG:

$S \Rightarrow \mathtt{a}T\mathtt{b}$ (using rule $S \rightarrow \mathtt{a}T\mathtt{b}$)

$\Rightarrow \mathtt{a}T\mathtt{ab}$ (using rule $T \rightarrow T\mathtt{a}$)

$\Rightarrow \mathtt{aab}$ (using rule $T \rightarrow \varepsilon$)

$$S \rightarrow \mathtt{a}T\mathtt{b} \mid \mathtt{b}$$
$$T \rightarrow T\mathtt{a} \mid \varepsilon$$



PDA Example

State	Input	Stack	Equiv Rule
q_{start}	aab		
q_{loop}	aab	$S\$$	
q_{loop}	aab	$\mathtt{a}T\mathtt{b}\$$	$S \rightarrow \mathtt{a}T\mathtt{b}$
q_{loop}	ab	$T\mathtt{b}\$$	
q_{loop}	ab	$T\mathtt{ab}\$$	$T \rightarrow T\mathtt{a}$
q_{loop}	ab	ab$	$T \rightarrow \varepsilon$
q_{loop}	b	b$	
q_{loop}		$	
q_{accept}			

Machine is **parsing:**

1. Start with (input) string,
2. Find **rules** that **generate** string

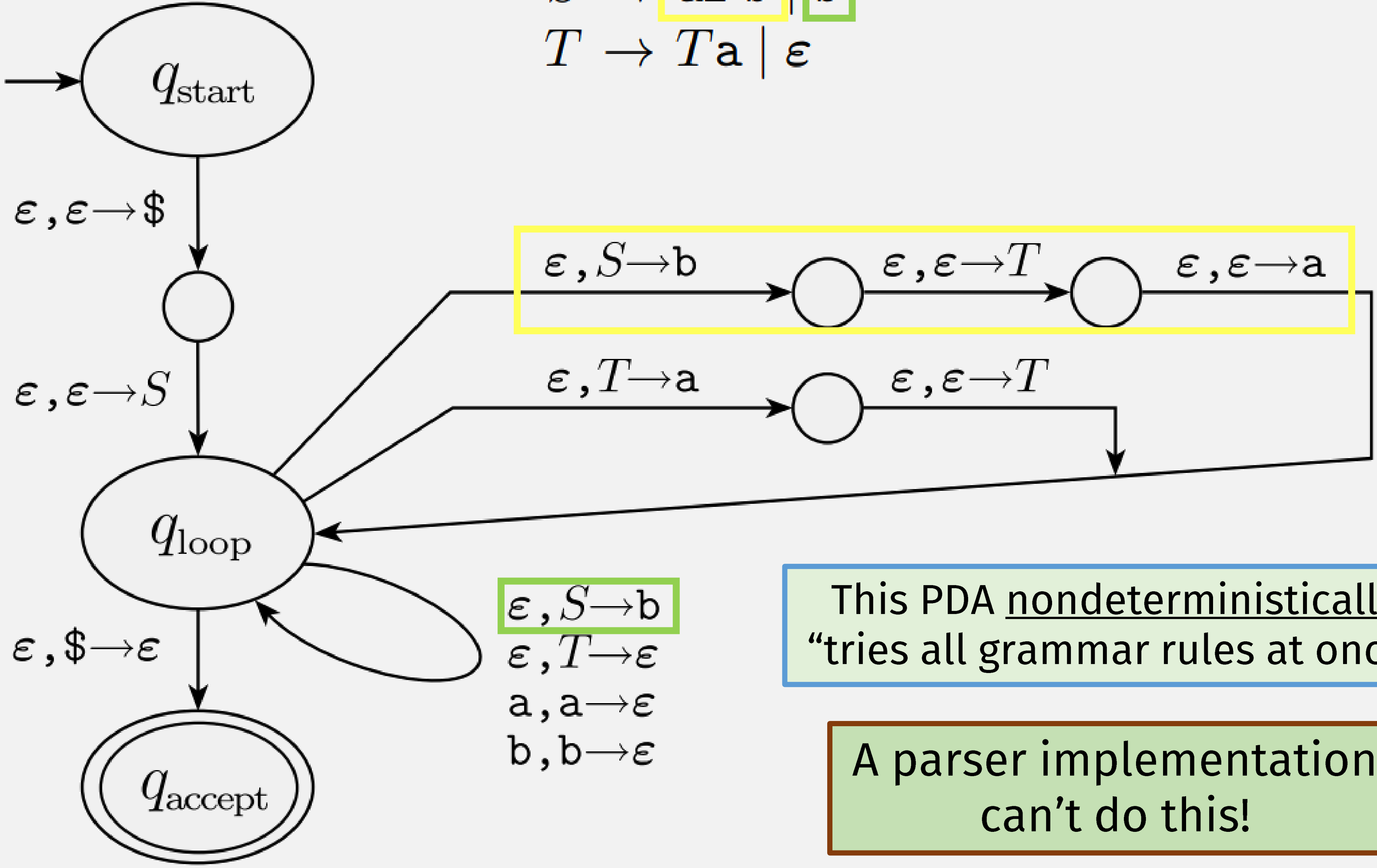
Generating vs Parsing

- In practice, **parsing** a string more important than **generating** one
  - E.g., a **compiler** (first step) **parses source code into a parse tree**
  - (Actually, *any* program with string inputs must first parse it)

- But: the PDAs we've seen are <u>non-deterministic</u> (like NFAs)

Previously: (Nondeterministic) PDA

$$S \rightarrow \boxed{\mathtt{a}T\mathtt{b}} \mid \boxed{\mathtt{b}}$$
$$T \rightarrow T\mathtt{a} \mid \varepsilon$$



This PDA nondeterministically "tries all grammar rules at once"

A parser implementation can't do this!

Generating vs Parsing

- In practice, **parsing** a string more important than **generating** one
  - E.g., a **compiler** (first step) **parses source code into a parse tree**
  - (Actually, *any* program with string inputs must first parse it)
- But: the PDAs we've seen are <u>non-deterministic</u> (like NFAs)
- Compiler's parsing algorithm must be <u>deterministic</u>
- <u>So</u>: to model parsers, we need a **Deterministic PDA** (DPDA)

DPDA: Formal Definition

The language of a DPDA is called a ***deterministic context-free language***.

A ***deterministic pushdown automaton*** is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, where Q, Σ, Γ, and F are all finite sets, and

1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. $\delta\colon Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longrightarrow \boxed{(Q \times \Gamma_\varepsilon) \cup \{\underbrace{\emptyset}_{\text{"do nothing"}}\}}$ is the transition function
5. $q_0 \in Q$ is the start state, and
6. $F \subseteq Q$ is the set of accept states.

A ***pushdown automaton*** is a 6-tuple

1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. $\delta\colon Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longrightarrow \boxed{\mathcal{P}}(Q \times \Gamma_\varepsilon)$
5. $q_0 \in Q$ is the start state, and
6. $F \subseteq Q$ is the set of accept states.

Difference: **DPDA has only one possible action,** for any given **state, input,** and **stack op** (similar to **DFA** vs **NFA**)

Must take into account ε reads or stack ops!
E.g., if $\delta(q, \mathrm{a}, X)$ **does "something",**
then $\delta(q, \varepsilon, X)$ **must "do nothing"**

DPDAs are Not Equivalent to PDAs!

- A PDA can non-deterministically "try all rules" (abandoning failed attempts);
- A DPDA must choose one rule at each step! (cant go back after reading input!)

$$R \to S \mid T$$
$$S \to \mathtt{a}S\mathtt{b} \mid \mathtt{ab}$$
$$T \to \mathtt{a}T\mathtt{bb} \mid \mathtt{abb}$$

Parsing = deriving reversed: start with **string**, end with **parse tree**

used S rule

aaabbb ⟿ aaSbb

When parsing this string, when does it know which rule was used, S or T?

used T rule

aaabbbbbb ⟿ aaTbbbb

Choosing "correct" rule depends on rest of the input!

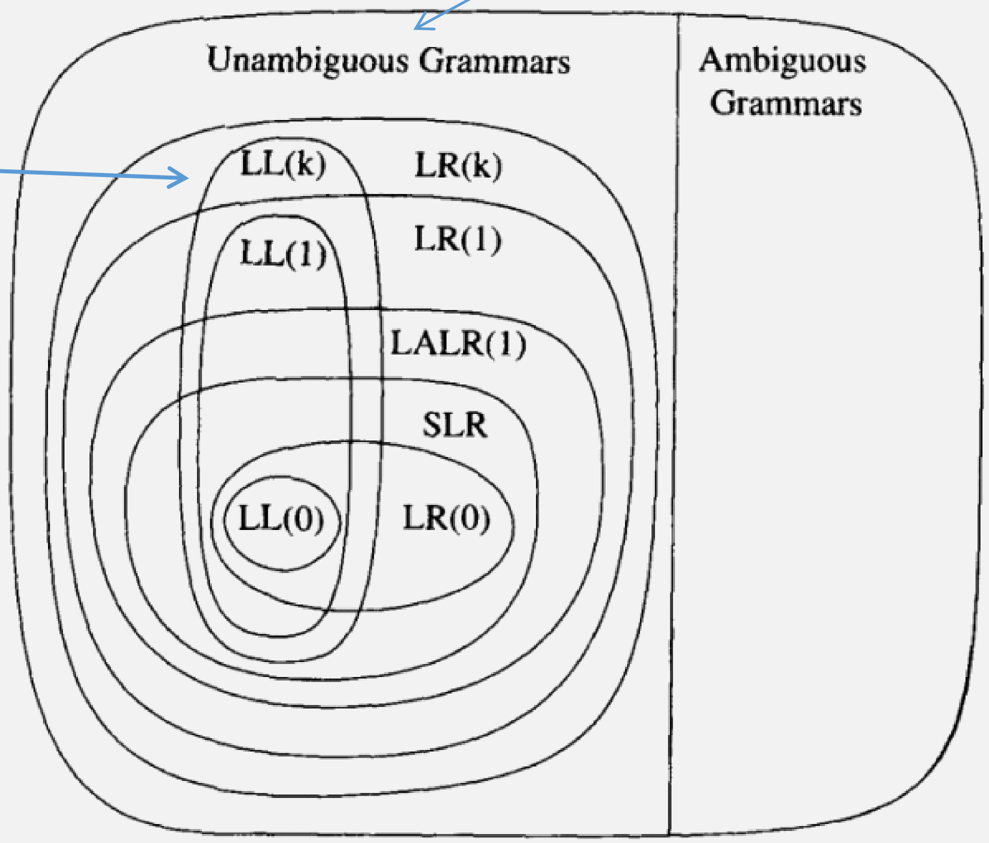
PDAs recognize CFLs, but **DPDAs only recognize DCFLs!** (a subset of CFLs)

Subclasses of CFLs

Umambiguous CFLs / PDAs

DCFLs

Programming language parsers / compilers are ideally in here



All CFLS

Compiler Stages

A program string (chars) (e.g., `a : = ( 5 + 3 ) ;` ...)

Lexer

DFAs (recognizing regular languages) in here!

Program "words"
(e.g., `ID(a) ASSIGN LPAREN NUM(5) PLUS NUM(3) RPAREN SEMI` ...)

A Lexer Implementation

```
%{
/* C Declarations: */
#include "tokens.h"    /* definitions of IF, ID, NUM, ... */
#include "errormsg.h"
union {int ival; string sval; double fval;} yylval;
int charPos=1;
#define ADJ  (EM_tokPos=charPos, charPos+=yyleng)
%}
/* Lex Definitions: */
digits   [0-9]+
%%
/* Regular Expressions and Actions: */
if                       {ADJ; return IF;}
[a-z][a-z0-9]*           {ADJ; yylval.sval=String(yytext);
                          return ID;}
{digits}                 {ADJ; yylval.ival=atoi(yytext);
                          return NUM;}
({digits}"."[0-9]*)|([0-9]*"."{digits})      {ADJ;
                          yylval.fval=atof(yytext);
                          return REAL;}
("--"[a-z]*"\n")|(" "|"\n"|"\t")+     {ADJ;}
.                        {ADJ; EM_error("illegal character");}
```

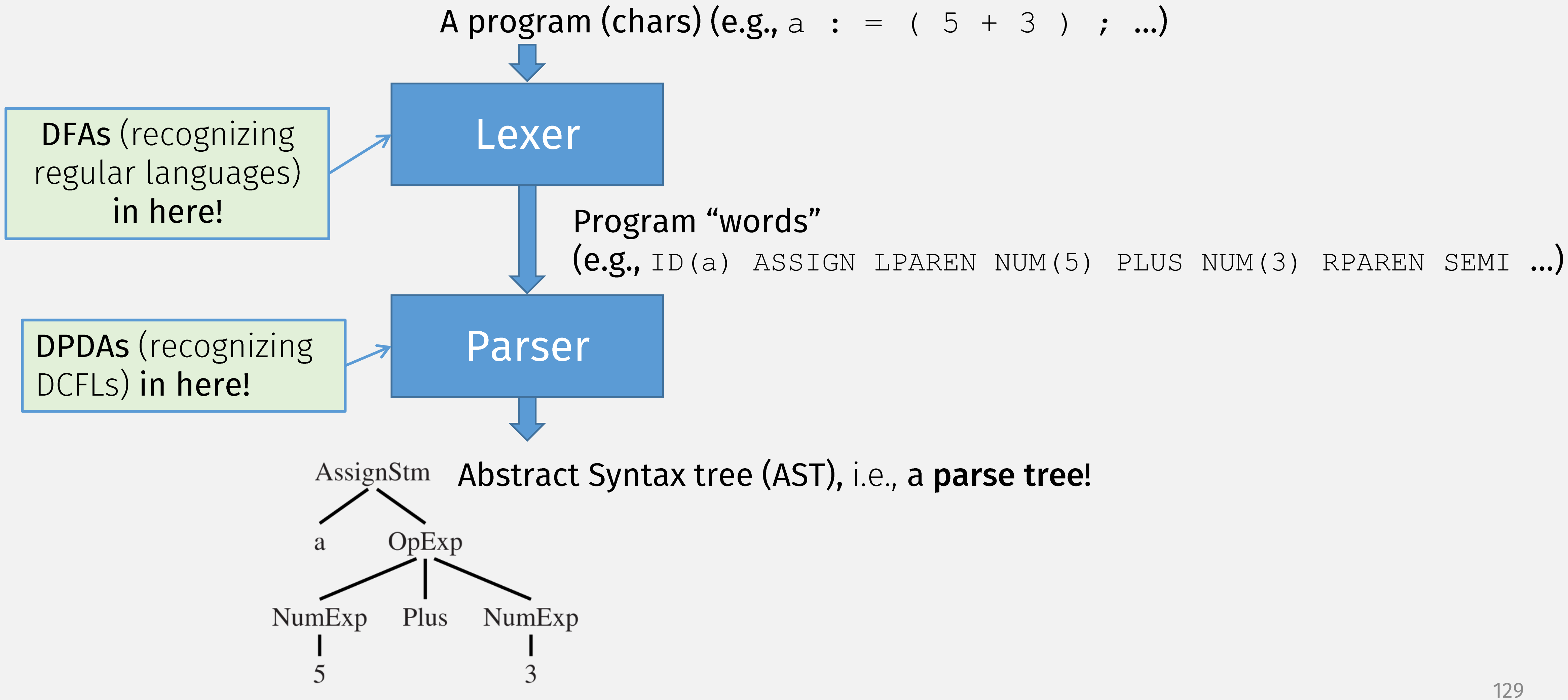
Remember our analogy:
- **DFAs** are like **programs**
- **All possible DFA tuples** is like **a programming language**

This DFA is a real program!

A "`lex`" tool converts the program:
- from "DFA Lang" …
- to an **equivalent** one in C !

DFAs (represented as **regular expressions**)!

Compiler Stages



A Parser Implementation

```
%{
int yylex(void);
void yyerror(char *s) { EM_error(EM_tokPos, "%s", s); }
%}
%token ID WHILE BEGIN END DO IF THEN ELSE SEMI ASSIGN
%start prog
%%

prog: stmlist

stm : ID ASSIGN ID
    | WHILE ID DO stm
    | BEGIN stmlist END
    | IF ID THEN stm
    | IF ID THEN stm ELSE stm

stmlist : stm
        | stmlist SEMI stm
```

Just write the CFG!

Remember our analogy:
CFGs are like **programs**

This CFG is a real program!

A "`yacc`" tool converts the program:
- from "CFG Lang" ...
- to an **equivalent** one in C !

DPDAs are Not Equivalent to PDAs!

Parsing = generating reversed:
- start with string
- end with parse tree

$$R \rightarrow S \mid T$$
$$S \rightarrow \boxed{aSb} \mid ab$$
$$T \rightarrow \boxed{aTbb} \mid abb$$

- **PDA:** can non-deterministically "try all rules" (abandoning failed attempts);
- **DPDA:** must choose one rule at each step!

Should use S rule

$$aaabbb \rightarrowtail aaSbb$$

aaa

Should use T rule

$$aaabbbbbb \rightarrowtail aaTbbbb$$

When parsing reaches this position, does it know which rule, S or T?

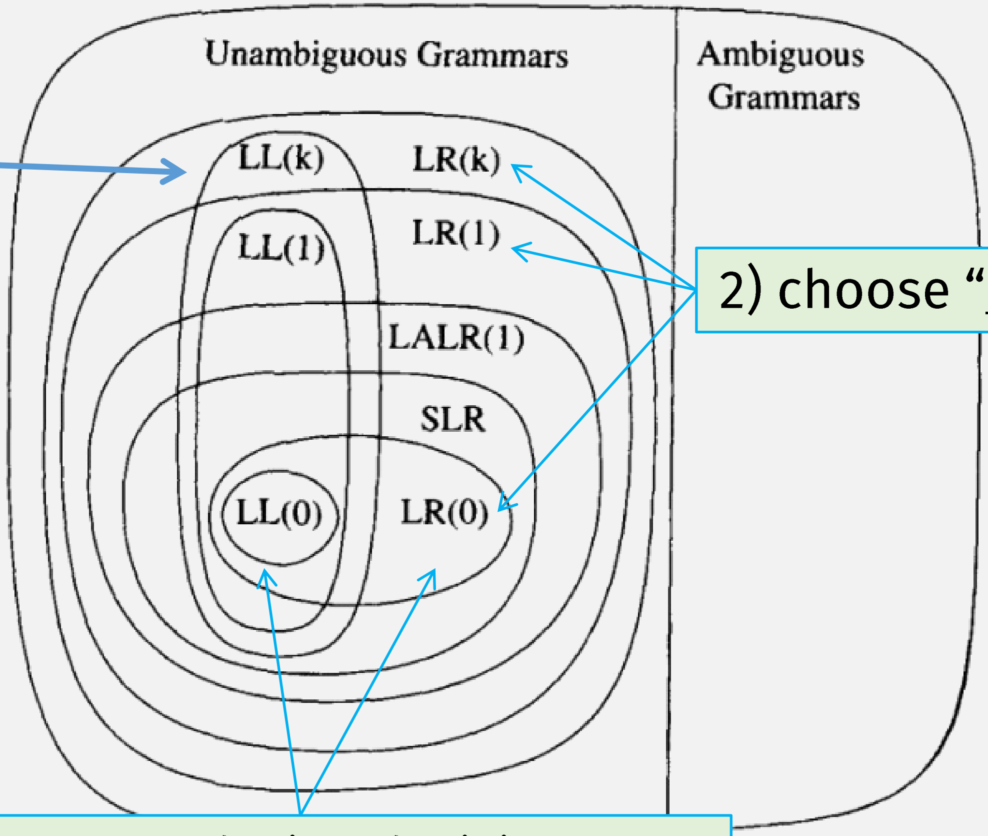
To choose "correct" rule, need to "look ahead" at rest of the input!

PDAs recognize CFLs, but **DPDAs only recognize DCFLs!** (a subset of CFLs)

Subclasses of CFLs

DCFLs

Programming language parsers / compilers are ideally in here



2) choose "look ahead" amount

2 parser design decisions:
1) Parse from left, or from right

All CFLS

LL parsing

Game: <u>"You're the Parser"</u>:
Guess which rule applies?

- **L** = left-to-right
- **L** = leftmost derivation

1 $S \rightarrow \boxed{\text{if } E \text{ then } S \text{ else } S}$

2 $S \rightarrow \text{begin } S\ L$

3 $S \rightarrow \text{print } E$

4 $L \rightarrow \text{end}$

5 $L \rightarrow ;\ S\ L$

6 $E \rightarrow \text{num} = \text{num}$

```
if 2 = 3 begin print 1; print 2; end else print 0
```

LL parsing

- **L** = left-to-right
- **L** = leftmost derivation

1 $S \rightarrow \text{if } E \text{ then } S \text{ else } S$

2 $S \rightarrow \text{begin } S\ L$

3 $S \rightarrow \text{print } E$

4 $L \rightarrow \text{end}$

5 $L \rightarrow ;\ S\ L$

6 $E \rightarrow \boxed{\text{num} = \text{num}}$

```
if 2 = 3 begin print 1; print 2; end else print 0
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LL parsing

- **L** = left-to-right
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1 $S \rightarrow \text{if } E \text{ then } S \text{ else } S$

2 $S \rightarrow \boxed{\text{begin } S\ L}$

3 $S \rightarrow \text{print } E$

4 $L \rightarrow \text{end}$

5 $L \rightarrow \text{; } S\ L$

6 $E \rightarrow \text{num } = \text{ num}$

```
if 2 = 3 begin print 1; print 2; end else print 0
```

LL parsing

- **L** = left-to-right
- **L** = leftmost derivation

1 $S \rightarrow \text{if } E \text{ then } S \text{ else } S$

2 $S \rightarrow \text{begin } S\ L$

3 $S \rightarrow \boxed{\text{print } E}$

4 $L \rightarrow \text{end}$

5 $L \rightarrow \ ;\ S\ L$

6 $E \rightarrow \text{num} = \text{num}$

```
if 2 = 3 begin print 1; print 2; end else print 0
```

"Prefix" languages (Scheme/Lisp) **are easily parsed with LL parsers** (zero lookahead)

LR parsing

- **L** = left-to-right
- **R** = rightmost derivation

1 $S \rightarrow S \,;\, S$

2 $\boxed{S \rightarrow \mathrm{id} := E}$

3 $S \rightarrow \mathrm{print}\ (\ L\ )$

4 $\boxed{E \rightarrow \mathrm{id}}$

5 $E \rightarrow \mathrm{num}$

6 $\boxed{E \rightarrow E + E}$

```
a := 7;
b := c + (d := 5 + 6, d)
```

When parse is here, can't determine whether it's an assign (**:=**) or addition (**+**)

Need to <u>save</u> input (lookahead) to some memory, like a **stack**! this is a job for a (D)PDA!

LR parsing

- **L** = left-to-right
- **R** = rightmost derivation

$$S \to S\ ;\ S \qquad E \to \text{id}$$
$$S \to \text{id} := E \qquad E \to \text{num}$$
$$S \to \text{print}\ (\ L\ ) \qquad E \to E + E$$

```
a := 7;
b := c + (d := 5 + 6, d)
```

Stack	*Input*	*Action*
1	a := 7 ; b := c + (d := 5 + 6 , d) $	*shift*

push

= “push”

State name

C

LR parsing

- **L** = left-to-right
- **R** = rightmost derivation

$$
\begin{array}{ll}
S \rightarrow S \,;\, S & E \rightarrow \mathrm{id} \\
S \rightarrow \mathrm{id} := E & E \rightarrow \mathrm{num} \\
S \rightarrow \mathrm{print}\ (\ L\ ) & E \rightarrow E + E
\end{array}
$$

Stack	*Input*	*Action*
$_1$	`a := 7 ; b := c + ( d := 5 + 6 , d ) $`	*shift*
$_1$ id$_4$	`:= 7 ; b := c + ( d := 5 + 6 , d ) $`	*shift*
$_1$ id$_4$:=$_6$	`7 ; b := c + ( d := 5 + 6 , d ) $`	*shift*

LR parsing

- **L** = left-to-right
- **R** = rightmost derivation

$$S \rightarrow S \,;\, S \qquad E \rightarrow \mathrm{id}$$
$$S \rightarrow \mathrm{id} := E \qquad E \rightarrow \mathrm{num}$$
$$S \rightarrow \mathrm{print}\ (\ L\ ) \qquad E \rightarrow E + E$$

Stack	*Input*	*Action*
$_1$	`a := 7 ; b := c + ( d := 5 + 6 , d ) $`	*shift*
$_1\ \mathrm{id}_4$	`:= 7 ; b := c + ( d := 5 + 6 , d ) $`	*shift*
$_1\ \mathrm{id}_4\ :=_6$	`7 ; b := c + ( d := 5 + 6 , d ) $`	*shift*
$_1\ \mathrm{id}_4\ :=_6\ \mathrm{num}_{10}$	`; b := c + ( d := 5 + 6 , d ) $`	*reduce* $E \rightarrow \mathrm{num}$

LR parsing

- **L** = left-to-right
- **R** = rightmost derivation

1. $S \rightarrow S\ ;\ S$
2. $S \rightarrow \mathrm{id} := E$
3. $S \rightarrow \mathrm{print}\ (\ L\ )$
4. $E \rightarrow \mathrm{id}$
5. $\boxed{E \rightarrow \mathrm{num}}$
6. $E \rightarrow E + E$

Stack	*Input*	*Action*
1	a := 7 ; b := c + (d := 5 + 6 , d) $	*shift*
$_1$ id$_4$	:= c + (d := 5 + 6 , d) $	*shift*
$_1$ id$_4$:=$_6$	:= c + (d := 5 + 6 , d) $	*shift*
$_1$ id$_4$:=$_6$ $\boxed{\mathrm{num}_{10}}$	; b := c + (d := 5 + 6 , d) $	*reduce* $E \rightarrow \mathrm{num}$

Can determine (rightmost) rule

LR parsing

- **L** = left-to-right
- **R** = rightmost derivation

1 $S \rightarrow S \; ; \; S$

2 $\boxed{S \rightarrow \text{id} := E}$

3 $S \rightarrow \text{print} \; (\; L \;)$

4 $E \rightarrow \text{id}$

5 $E \rightarrow \text{num}$

6 $E \rightarrow E + E$

Stack	*Input*	*Action*
$_1$	a := 7 ; b := c + (d := 5 + 6 , d) \$	*shift*
$_1 \text{ id}_4$	:= 7 ; b := c + (d := 5 + 6 , d) \$	*shift*
$_1 \text{ id}_4 \; :=_6$	= c + (d := 5 + 6 , d) \$	*shift*
$_1 \text{ id}_4 \; :=_6 \text{ num}_{10}$	= c + (d := 5 + 6 , d) \$	*reduce* $E \rightarrow \text{num}$
$_1 \text{ id}_4 \; :=_6 \; E_{11}$	; b := c + (d := 5 + 6 , d) \$	*reduce* $S \rightarrow \text{id} := E$

Can determine (rightmost) rule

LR parsing

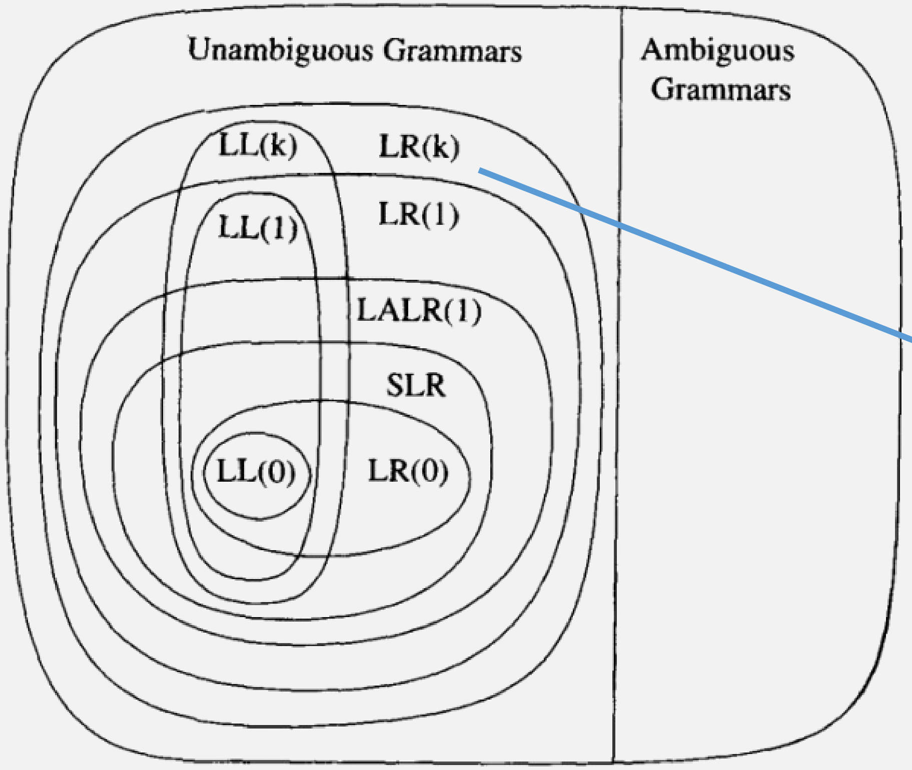
- **L** = left-to-right
- **R** = rightmost derivation

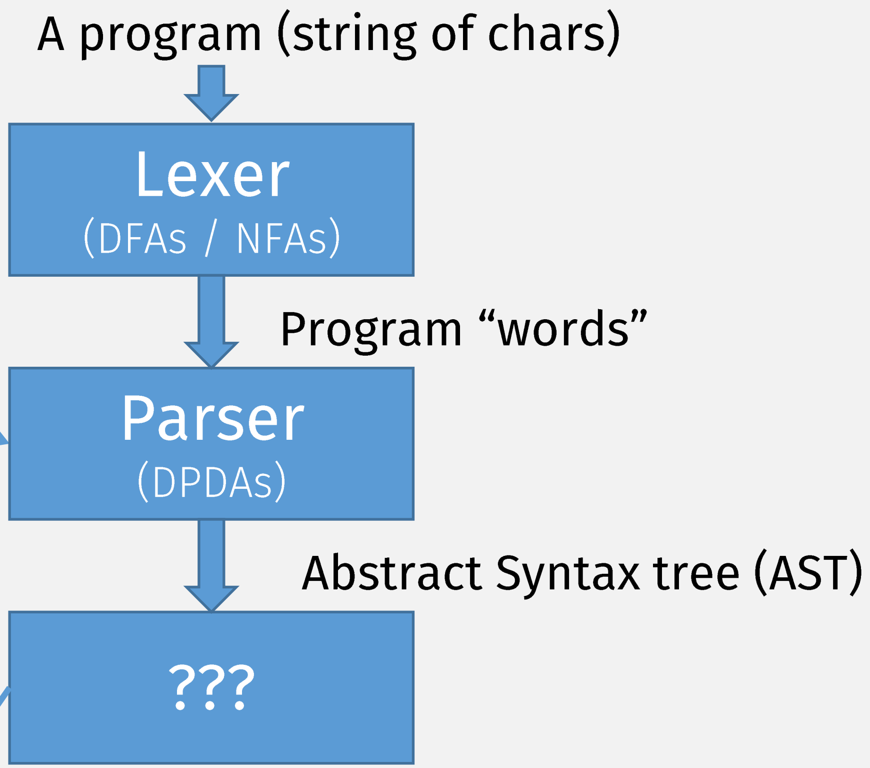
$$S \to S \,;\, S \qquad E \to \mathrm{id}$$
$$\boxed{S \to \mathrm{id} := E} \qquad E \to \mathrm{num}$$
$$S \to \mathrm{print}\,(\,L\,) \qquad E \to E + E$$

Stack	*Input*	*Action*
1	a := 7 ; b := c + (d := 5 + 6 , d) $	*shift*
1 id_4	:= 7 ; b := c + (d := 5 + 6 , d) $	*shift*
1 id_4 $:=_6$	7 ; b := c + (d := 5 + 6 , d) $	*shift*
1 id_4 $:=_6$ num_{10}	; b := c + (d := 5 + 6 , d) $	*reduce* $E \to \mathrm{num}$
1 $\boxed{\mathrm{id}_4 \; :=_6 \; E_{11}}$	; b := c + (d := 5 + 6 , d) $	*reduce* $S \to \mathrm{id} := E$
1 $\boxed{S_2}$	; b := c + (d := 5 + 6 , d) $	*shift*

LR Parsers also called "Shift-Reduce" Parsers

To learn more, take a Compilers Class!





This phase needs computation that goes beyond CFLs

In-class quiz 3/20

See gradescope