UMB CS 420

Non-CFLs

Wednesday, March 22, 2023

(AN UNMATCHED LEFT PARENTHESIS CREATES AN UNRESOLVED TENSION THAT WILL STAY WITH YOU ALL DAY.

Announcements

- HW 6
  - Due Sunday 3/26 11:pm EDT

Quiz Preview

- The **Pumping Lemma for CFLs** states that:
  - all strings in a CFL that are longer than the pumping length can be split into 5 substrings $uvxyz$ …
  - … where repeating some of these substrings (together) results in a "pumped" string that is still in the language.
  - Which are the substrings that can be pumped (together) in this way?

Flashback: Pumping Lemma for Regular Langs

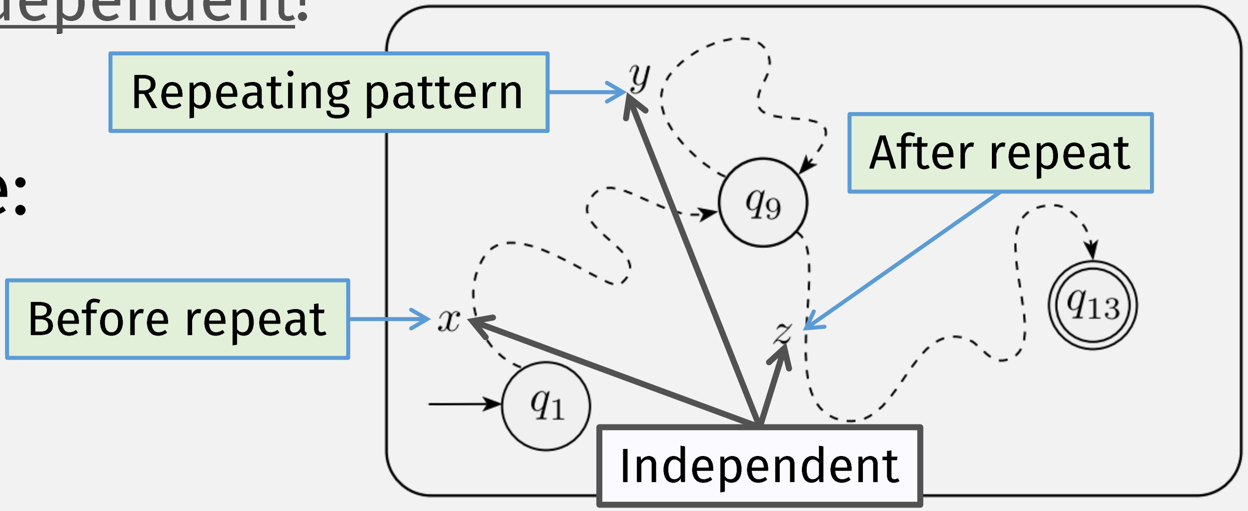
- **Pumping Lemma** describes how strings repeat

- **Regular language** strings repeat using **Kleene star** operation
  - 3 substrings $x\ y\ z$ are independent!

- A non-regular language:

$$\{0^n 1^n \mid n \geq 0\}$$

Kleene star can't express this pattern:
2nd part depends on (length of) 1st part



- Q: How do CFLs repeat?

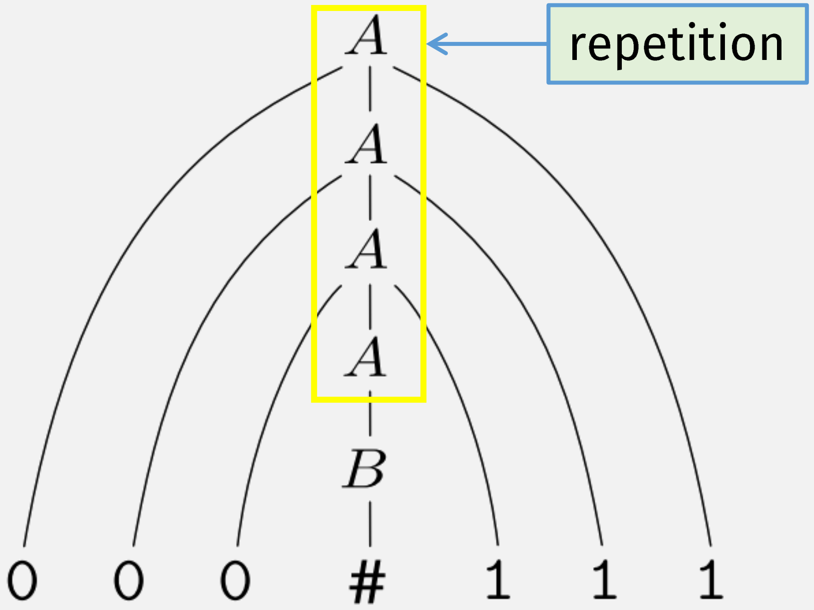
Repetition and Dependency in CFLs

Parts before/after repetition point are linked

Repetition

$$A \rightarrow 0A1$$
$$A \rightarrow B$$
$$B \rightarrow \#$$

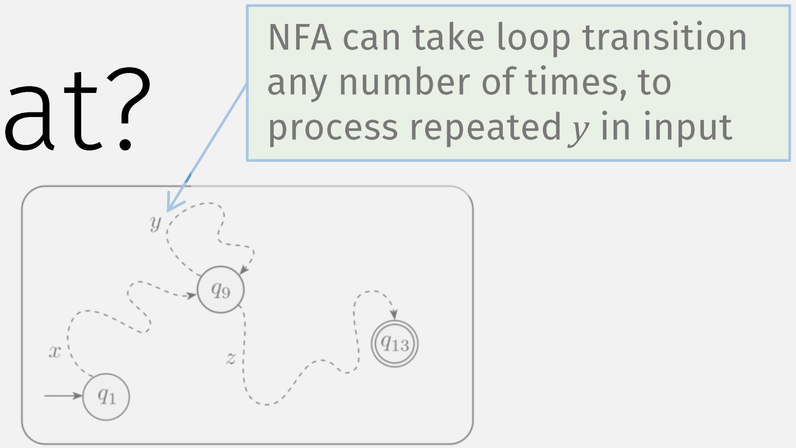
$$\{0^n\#1^n \mid n \geq 0\}$$



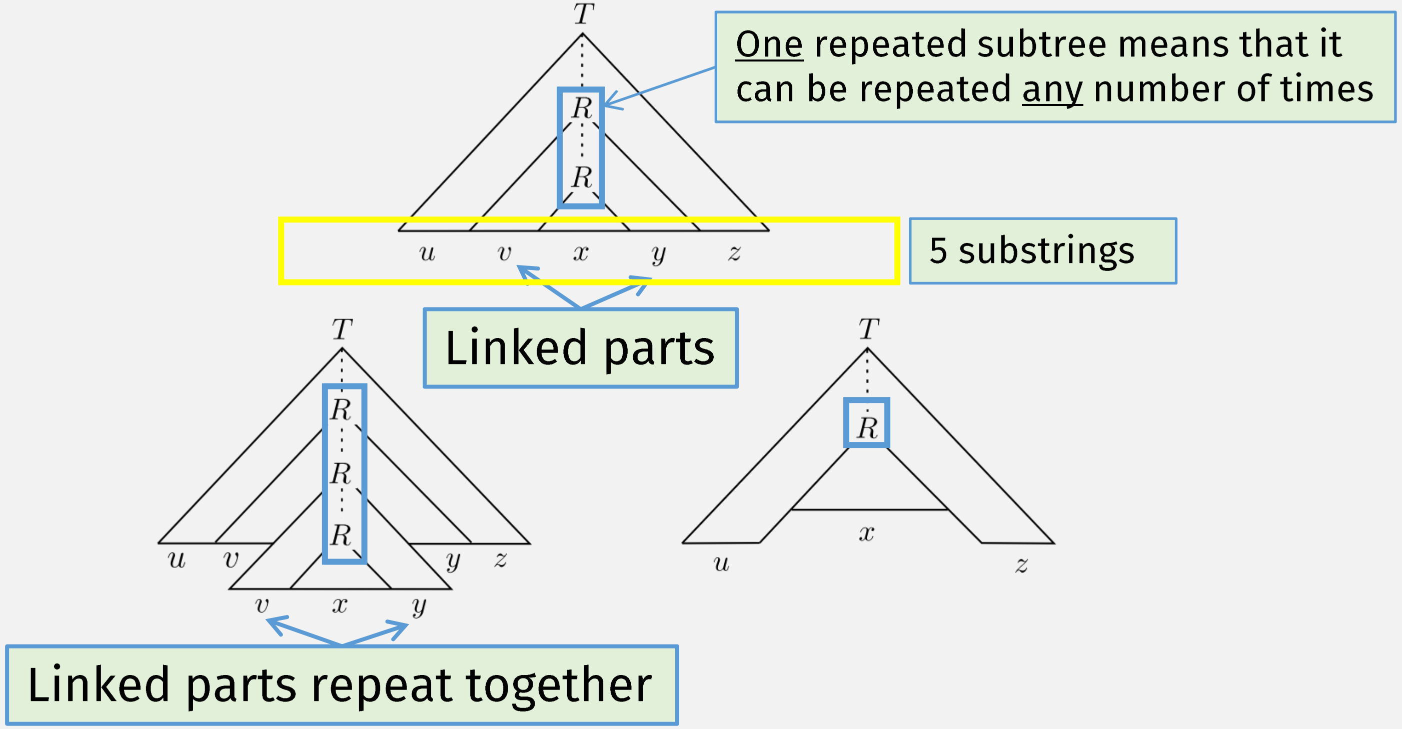
$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\#111$$

How Do Strings in CFLs Repeat?

- Strings in regular languages repeat states



- Strings in CFLs repeat subtrees in the parse tree

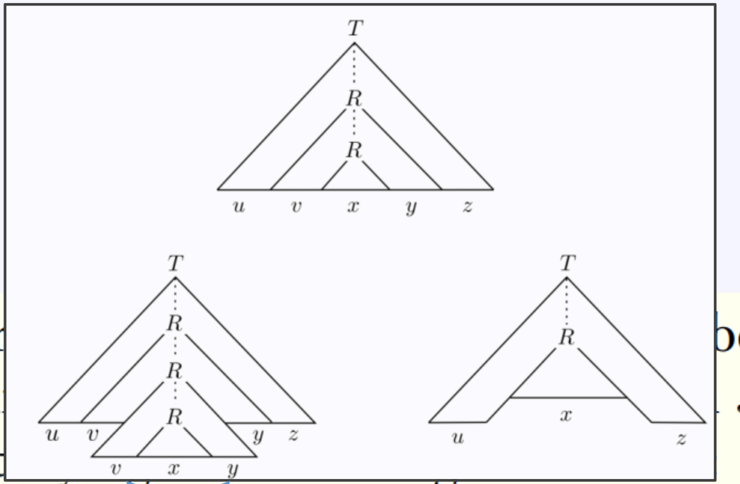


Pumping Lemma for CFLS

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

Two pumpable parts.
But they must be pumped together!

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.



Pumping lemma If A is a regular ... ber p (the pumping length) where if s is any stri... s may be divided into three pieces, $s = xyz$, sat...

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

One pumpable part

Two pumpable parts, pumped together

A Non CFL example

$$\text{language } B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\} \text{ is not context free}$$

Intuition

- Strings in CFLs can have two parts that are "pumped" together
- This language requires three parts to be "pumped" together
- So it's not a CFL!

Proof?

Want to prove: $a^n b^n c^n$ **is not** a CFL

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^i xy^i z \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

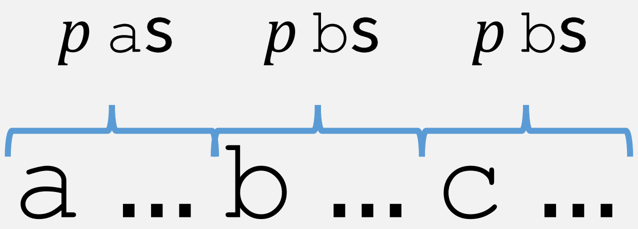
Reminder: CFL Pumping lemma says: all strings $a^n b^n c^n \geq$ length p are splittable into $uvxyz$ where v and y are pumpable

Proof (by contradiction):

Now we must find a contradiction …

- Assume: $a^n b^n c^n$ **is** a CFL
  - So it must satisfy the pumping lemma for CFLs
  - I.e., all strings $\geq$ length p are pumpable
- Counterexample = $a^p b^p c^p$

Contradiction if: string $\geq$ length p that is **not splittable** into $uvxyz$ where v and y are pumpable



Want to prove: $a^n b^n c^n$ **is not** a CFL

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Possible Splits

Proof (by contradiction):

contradiction

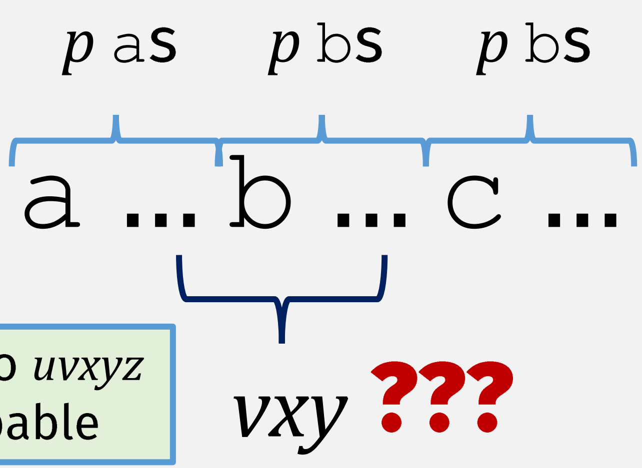
- Assume: $a^n b^n c^n$ **is** a CFL
  - So it must satisfy the pumping lemma for CFLs
  - I.e., all strings ≥ length p are pumpable
- Counterexample = $a^p b^p c^p$

Contradiction if: string ≥ length p that is **not splittable** into $uvxyz$ where v and y are pumpable

- Possible Splits (using condition # 3: $|vxy| \leq p$)

Not pumpable

- ☒ vxy is all as
- ☒ vxy is all bs
- ☒ vxy is all cs
- ☒ vxy has as and bs
- ☒ vxy has bs and cs



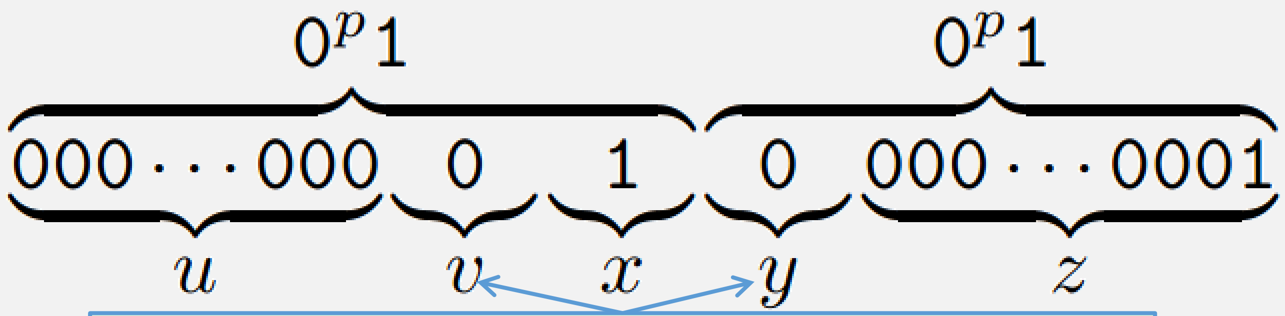
$a^p b^p c^p$ **cannot be split** into $uvxyz$ where v and y are pumpable

So $a^n b^n c^n$ **is not a CFL**
(justification:
contrapositive of CFL pumping lemma)

Another Non-CFL $D = \{ww \mid w \in \{0,1\}^*\}$

Be careful when choosing **counterexample** s: 0^p10^p1

This s can be pumped according to **CFL pumping lemma:**



Pumping v and y (together) produces string still in D

- CFL Pumping Lemma conditions:
  - ☑ **1.** for each $i \geq 0$, $uv^ixy^iz \in A$,
  - ☑ **2.** $|vy| > 0$, and
  - ☑ **3.** $|vxy| \leq p$.

This doesn't prove that the language is a CFL! It only means that this attempt to prove that the language is not a CFL failed.

Another Non-CFL $D = \{ww \mid w \in \{0,1\}^*\}$

- Need another counterexample string *s*:

If *vyx* is contained in first or second half, then any pumping will break the match ✖

$$\overbrace{0^p 1^p}\overbrace{0^p 1^p}$$

$$0^p\underbrace{1^p 0^p}1^p$$

So *vyx* must straddle the middle ✖

But any pumping still breaks the match because order is wrong

- CFL Pumping Lemma conditions:

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Now we have proven that this language is **not a CFL!**

A Practical Non-CFL

- **XML**
  - ELEMENT → <TAG>CONTENT</TAG>
  - Where TAG is any string

- XML also looks like this non-CFL: $D = \{ww \mid w \in \{0,1\}^*\}$

- This means XML is not context-free!
  - Note: HTML *is* context-free because …
  - … there are only a finite number of tags,
  - so they can be embedded into a finite number of rules.

In practice:

- XML is parsed as a CFL, with a CFG
- Then matching tags checked in a 2nd pass with a more powerful machine …

Next: A More Powerful Machine …

M_1 accepts its input if it is in language: $B = \{w\#w \mid w \in \{0,1\}^*\}$

M_1 = "On input string w:

1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether these positions contain the same symbol. If they do not, or if no # is found, *reject*. Cross off symbols as they are checked to keep track of which symbols correspond.

Infinite memory (initial contents are the **input string**)

Can move to, and **read/write from** <u>arbitrary</u> **memory locations!**

In-class quiz 3/22

See gradescope