Welcome to CS420!

Intro to Theory of Computation

UMass Boston Computer Science

Instructor: Stephen Chang

Spring 2024

Today's Theme:
What's CS 420 about?

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Intro to Theory of **Computation**

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What's this?

Interlude: CS 420 Lecture Logistics

- *I expect:* **lecture** to be **interactive**
  - **Participation** is a part of your **grade**
  - Also, **it's the best way to learn!**

- *I may:* **call** on **students randomly**
  - It's **ok to be wrong in class**! – will not affect your grade
  - Also, **it's the best way to learn!**

- *Please:* **tell me your name** before **speaking**
  - Sorry in advance if I get it wrong
  - Also, **it's the best way for me to learn!**

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Intro to Theory of Computation

How would you define this?

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Computation Is … (via examples)

- 1 + 1 = ??
- = 2

… some **basic** **<u>definitions</u>** and **<u>assumptions</u>** (**“axioms”**),
e.g., **define “Numbers” (in order)** to be**: 0, 1, 2, 3, …** **…**

- 11 + 11 = ??
- = 22

… and **<u>rules</u>** that use the **definitions** and **axioms** (**“algorithm”**),
e.g., grade school arithmetic

- 9999999999 + 9999999999 = ??
- = 19999999998

Computation rules can be <u>executed</u> by
hand, or by **machine / automaton**

- 1 +1 = ??
- = 10

(binary)

There are <u>many</u> **possible definitions**
(models) of **computation**

(hint)

Computation Is … Programs!

Every programming language is a model of computation

```
def f(x):
    if x > 0:
        return x + 1
    else:
        return x - 1
```

→ 11

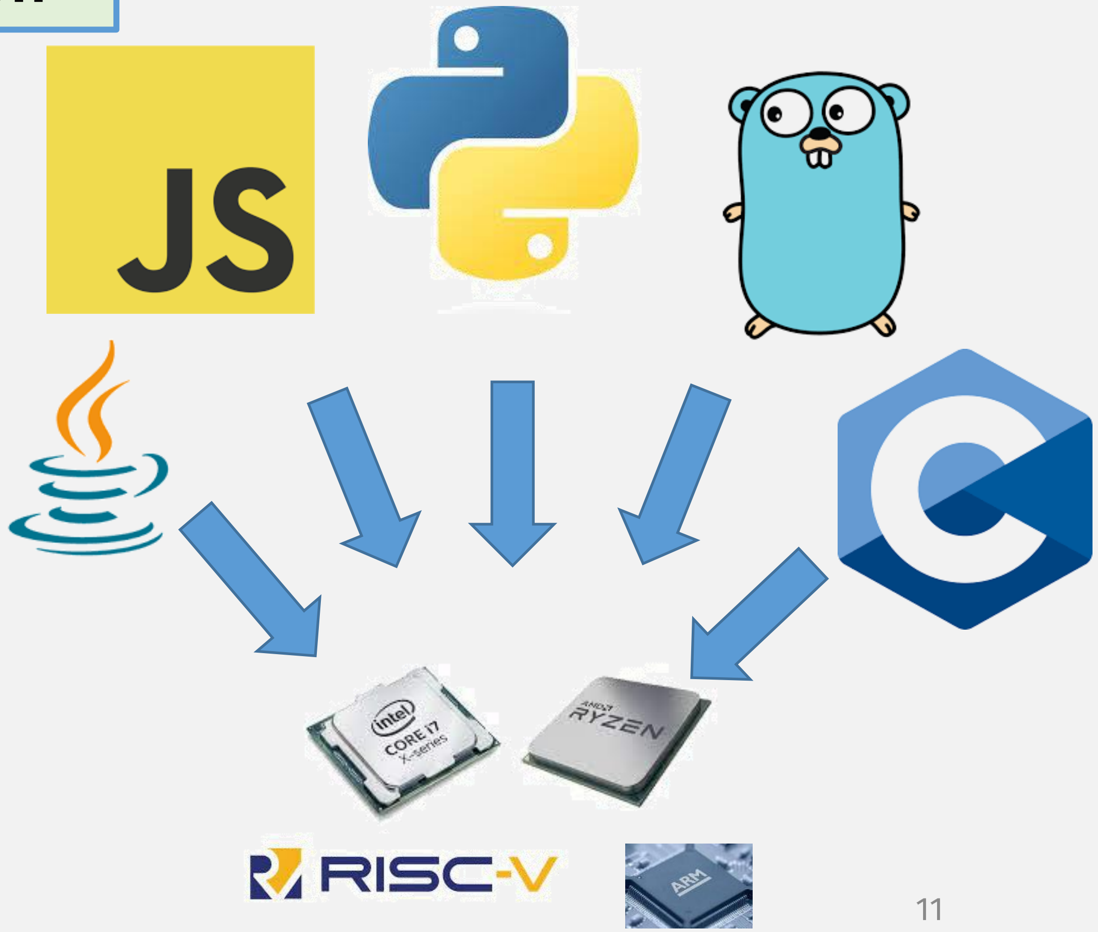
different???

If they are different: **how can we know?**

Or same???

If they are the same: **what is a** (simple) **model for all of them**

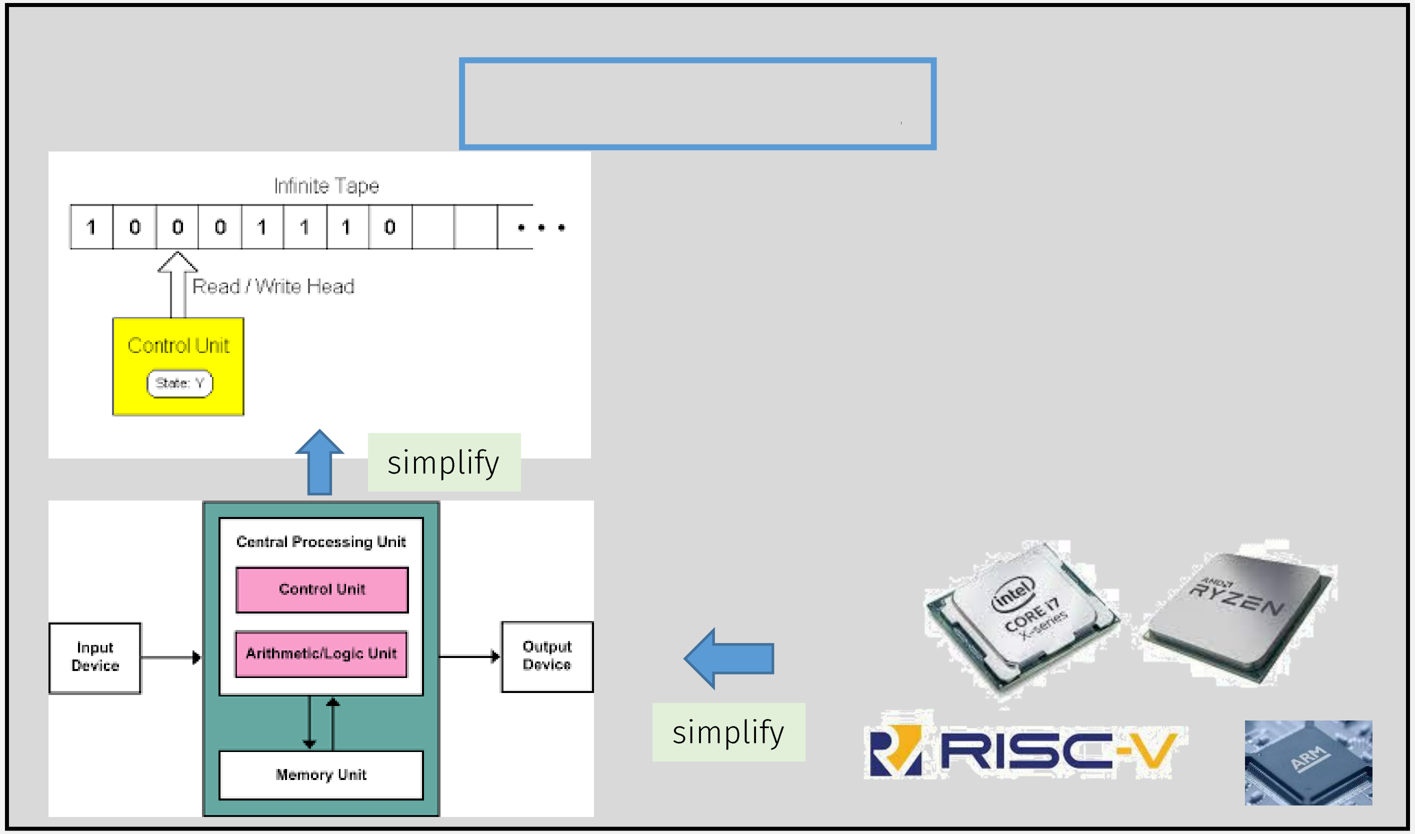
You already use **models of computation!**



In CS 420 this semester, we will …

1. Define and study **models of computation**
    - models will be *as simple as possible* (to make them easier to study)

Models of Computation



In CS 420 this semester, we will …

1. Define and study **models of computation**
   - **models** will be *as simple as possible* (to make them easier to study)

2. Compare & contrast models of computation
   - which "programs" are *included* by a model
   - which "programs" are *excluded* by a model
   - *overlap* between models?

Models of Computation

Turing Machines

Q: Are there computational models … other than **Turing Machines?**

Q: Are there computational models … “*weaker*” than **Turing Machines?**

Q: Are there computational models … “*more powerful*” than **Turing Machines?**

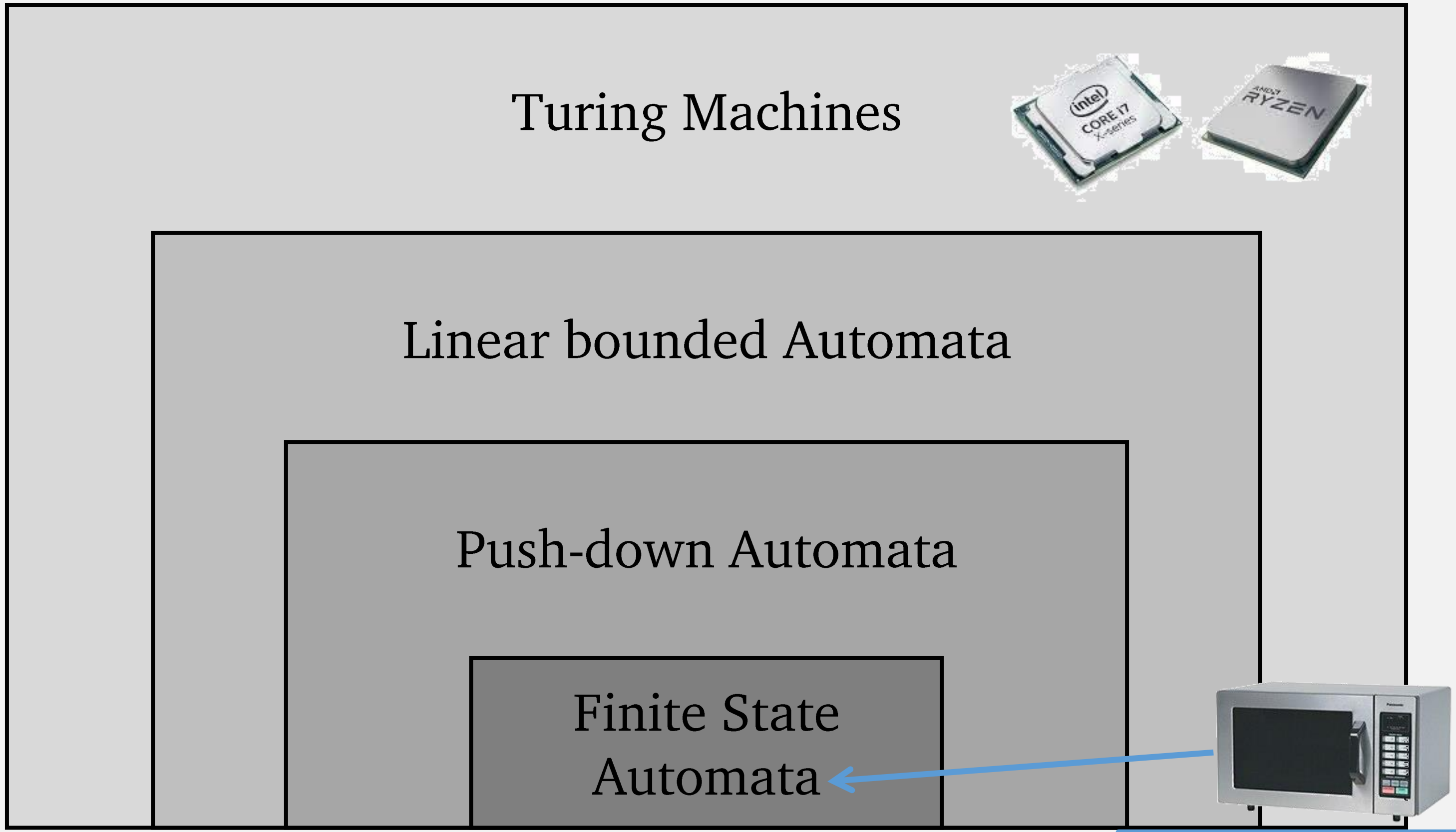
Q: What does “*weaker*” or “*more powerful*” even mean?!

A: Yes!

Models of Computation Hierarchy

... and get to here ...

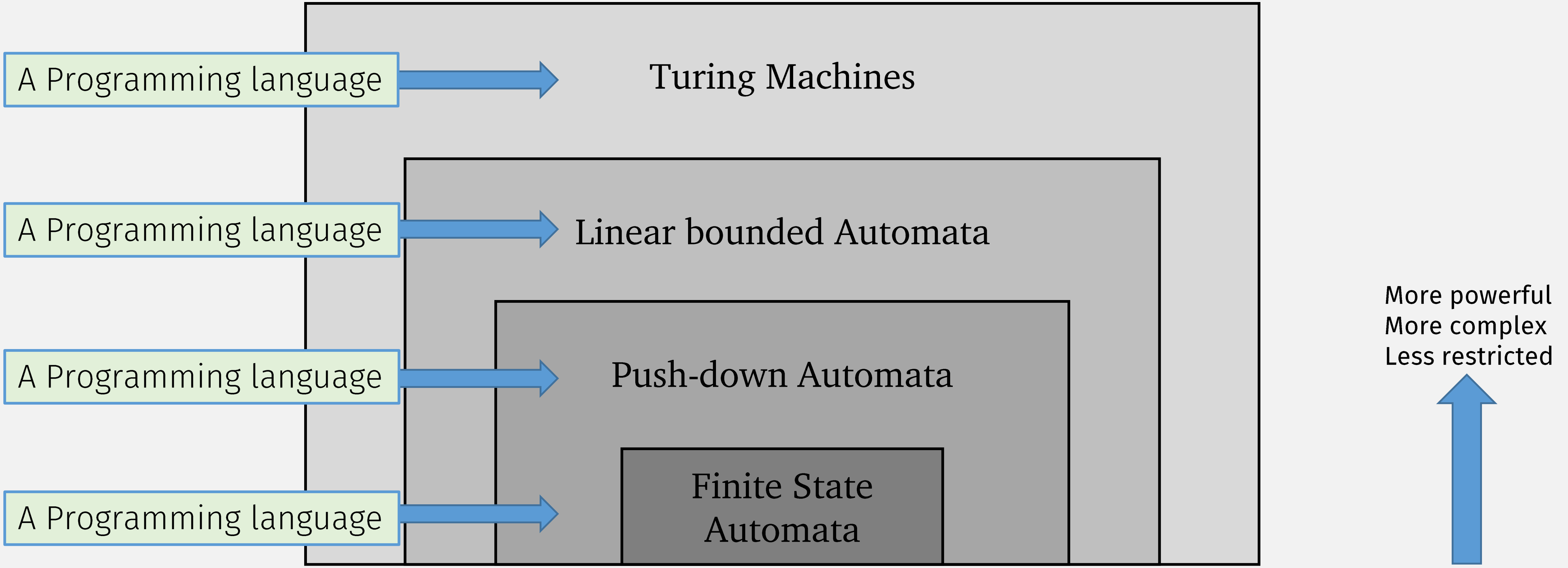
... and also look at what's out here???



We'll start here ...

More powerful
More complex
Less restricted

But remember … Computation = Programs!



Helpful analogy for this course:

- a **class of machines** (each rectangle above) ~ a **Programming Language**!
- a **single machine** (one thing in a rectangle) ~ a **Program**!

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What's this? ☑

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Intro to **Theory** of Computation

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What's this?

"Theory" = math

(This is a math course!)

(But programming is math too!)

Programming Is (What) Math?

Math(ematical) logic!

```
def f(x):
    if x > 0:
        return x + 1
    else:
        return x − 1

print( f(10) )
```

???

→ 11

How did you figure out the answer?

(But programming is math too!)

Programming = Mathematical logic!

- “logic is the foundation of all computer programming”
  - https://www.technokids.com/blog/programming/its-easy-to-improve-logical-thinking-with-programming/

- “logic is the fundamental key to becoming a good developer”
  - https://www.geeksforgeeks.org/i-cant-use-logic-in-programming-what-should-i-do/

- “Analytical skill and logical reasoning are prerequisites of programming because coding is effectively logical problem solving at its core”
  - https://levelup.gitconnected.com/the-secret-weapon-of-great-software-engineers-22d57f427937

Programming = Mathematical logic!

Programming Concepts
- Functions
- Variables
- If-then
- Recursion
- Strings
- Sets (and other data structures)

Math(ematical Logic) Concepts
- Functions
- Variables
- If-then (implication)
- Recursion
- Strings
- Sets (and other groupings of data)

In CS 420 this semester, we will …

1. Define and study **models of computation**
   - **models** will be *as simple as possible* (to make them easier to study)

2. Compare & contrast models of computation
   - which "programs" are *included* by a model
   - which "programs" are *excluded* by a model
   - *overlap* between models?

3. Prove things about the models

You already do “Proof” when Programming

```
def f(x):
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1
    else:
        return 1 / 0

print( f(10) )
```

???

→
```
11
```

Can this function ever throw `ZeroDivisionError`?

How did you figure out the answer?

You did a proof!

(Let’s write it out formally)

A (Mathematical) Theory Is …

Mathematical theory

From Wikipedia, the free encyclopedia

A **mathematical theory** is a mathematical model of a branch of mathematics that is based on a set of axioms. It can also simultaneously be a body of knowledge (e.g., based on known axioms and definitions), and so in this sense can refer to an area of mathematical research within the established framework.[1][2]

Explanatory depth is one of the most significant theoretical virtues in mathematics. For example, set theory has the ability to systematize and explain number theory and geometry/analysis. Despite the widely logical necessity (and self-evidence) of arithmetic truths such as 1<3, 2+2=4, 6-1=5, and so on, a theory that just postulates an infinite blizzard of such truths would be inadequate. Rather an adequate theory is one in which such truths are derived from explanatorily prior axioms, such as the Peano Axioms or set theoretic axioms, which lie at the foundation of ZFC axiomatic set theory.

The singular accomplishment of axiomatic set theory is its ability to give a foundation for the derivation of the entirety of classical mathematics from a handful of axioms. The reason set theory is so prized is because of its explanatory depth. So a mathematical theory which just postulates an infinity of arithmetic truths without explanatory depth would not be a serious competitor to Peano arithmetic or Zermelo-Fraenkel set theory.[3][4]

… a mathematical model, i.e., **axioms** and **definitions**, of some domain, e.g. computers …

… that **explains** (**predicts**) some real-world phenomena …

… and can **derive** (prove) additional results (**theorems**) …

How Mathematics Works

More Theorems

More Axioms

More Definitions

Theorem

Theorem

Axioms

Definitions

Mathematician
(or student)

Actually, **it's not always so easy to create the next level …**
Preciseness is important
(just like in programming)

Proofs = Figuring out how to (precisely) **stack the pieces together**

The “Modus Ponens” Inference Rule

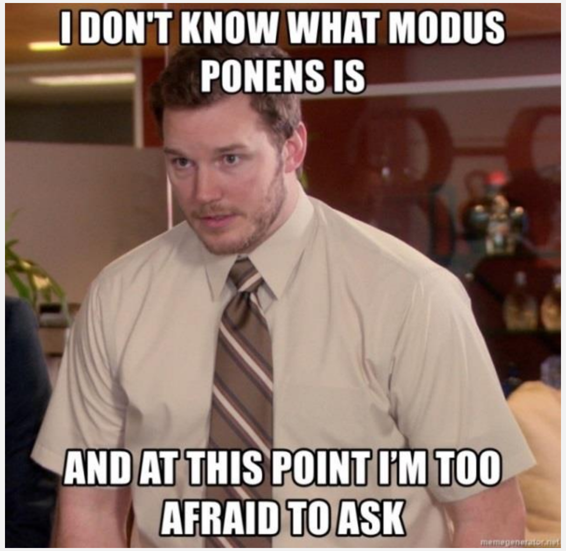
(Precisely Fitting Blocks Together)

Premises (if we can show these statements are true)

- If P then Q
- P is TRUE

Conclusion (then we can say that this is also true)

- Q must also be TRUE



Kinds of Mathematical Proof

Deductive Proof

- *Start with*: <u>known</u> facts and statements

- *Use:* logical **inference rules** (like modus ponens) to **prove** <u>new</u> **facts** and **statements**

Deductive Proof Example

```
def f(x):        "test expr"
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1  "first branch"
    else:
        return 1 / 0  "second branch"
```

Prove: fn f never throws `ZeroDivisionError`

Statements / Justifications Table

Proof:

Prior steps are already-proved, can be used to prove later steps!

Statements	Justifications
1. If running "test expr" is `True`, then "first branch" runs	1. Rules of Python
2. If running "test expr" is `False`, then "second branch" runs	2. Rules of Python
3. running "test expr" is (always) `True`	3. Definition of "numbers"
4. "first branch" (always) runs	4. By steps 1, 3, and modus ponens
7. fn f never throws `ZeroDivisionError`	

Modus Ponens

If we can prove these:

- If P then Q
- P

Then we've proved:

- Q

Deductive Proof Example

```
def f(x):
    if (x > 0) | (x < 0) | (x == 0):
        return x + 1 “first branch”
    else:
        return 1 / 0 “second branch”
```

Prove: fn f never throws ZeroDivisionError

Statements / Justifications Table

Proof:

Statements	**Justifications**
1. If running “test expr” is True, then “first branch” runs	1. Rules of Python
2. If running “test expr” is False, then “second branch” runs	2. Rules of Python
3. running “test expr” is (always) True	3. Definition of “numbers”
4. “first branch” (always) runs	4. By steps **1**, **3**, and modus ponens
5. “second branch” *never* runs	5. By steps **1**, **2**, and ???
6. fn f never runs 1 / 0	6. By step **5**
7. fn f never throws ZeroDivisionError	7. By step **6** and **???**