

**Welcome to CS420!**

# **Intro to Theory of Computation**

UMass Boston Computer Science

Instructor: Stephen Chang

Wednesday, January 24, 2024

Lecture 2

*Last Time*

**Welcome to CS420!**

# Intro to Theory of **Computation**

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*Analogy:*

**Computation Model**

(system of definitions and rules)



**Programming Language**

Last Time

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# Intro to Theory of Computation

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**“Theory” = math**  
(This is a math course!)

(But programming is math too!)

*And*  
A precisely defined

Computation Model  
(system of definitions and rules)



Programming Language

# Programming = Mathematical logic!

- “logic is the foundation of all computer programming”

- <https://www.technokids.com/blog/programming/its-easy-to-improve-logical-thinking-with-programming/>

- “logic is the fundamental key to becoming a good developer”

- <https://www.geeksforgeeks.org/i-cant-use-logic-in-programming-what-should-i-do/>

- “Analytical skill and logical reasoning are prerequisites of programming because coding is effectively logical problem solving at its core”

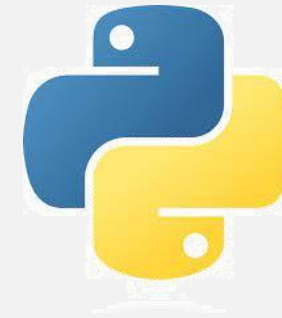
- <https://levelup.gitconnected.com/the-secret-weapon-of-great-software-engineers-22d57f427937>

# In CS 420 this semester, we will ...

1. Formally define and **study models of computation**
  - models will be as *simple* as possible (to make them easier to study)
2. Compare & contrast models of computation
  - which “programs” are *included* / *excluded* by a model
  - *Equality* or *overlap* between models?
3. Prove things about the models

# You already do “Proof” when Programming

```
def f(x):  
    if (x > 0) | (x < 0) | (x == 0):  
        return x + 1  
    else:  
        return 1 / 0
```



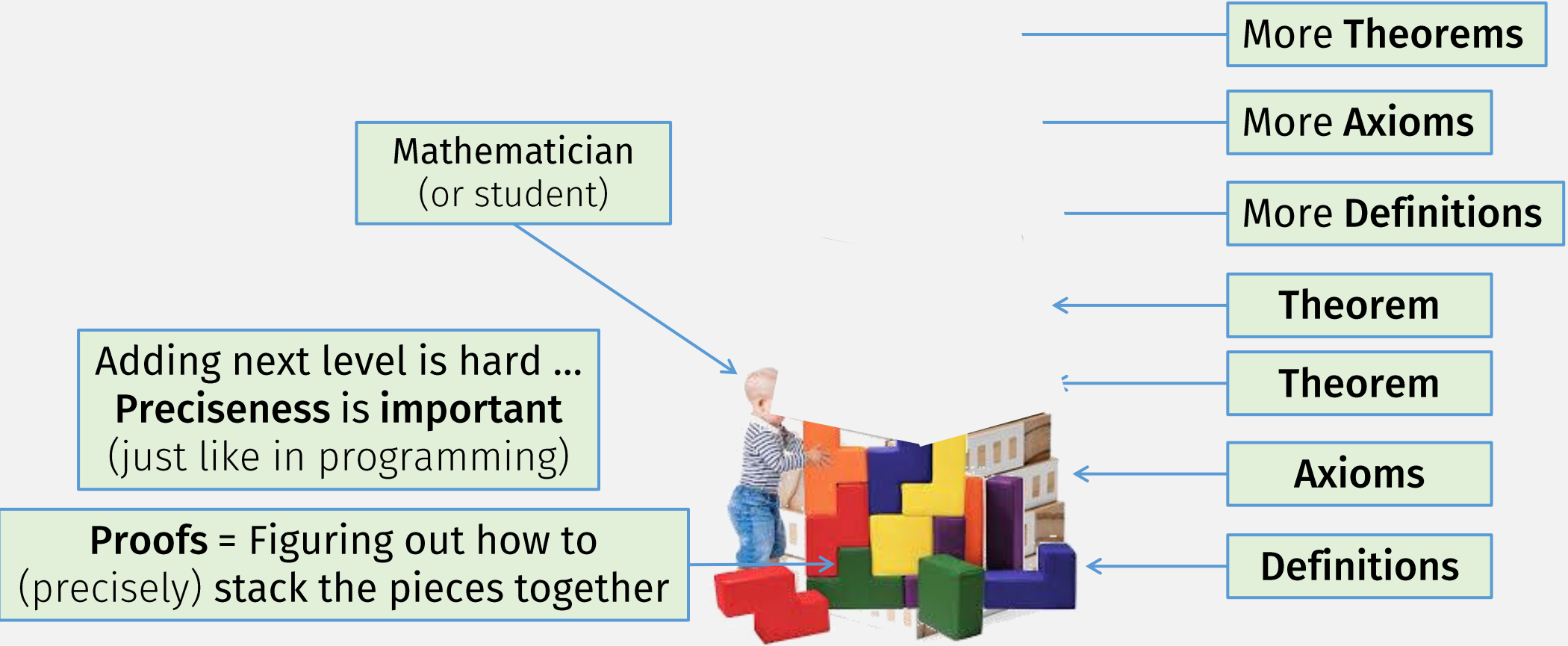
Can this function ever throw ZeroDivisionError?

How did you figure out the answer?

You did a proof!

(Let's write it out formally)

# How Mathematics (Proofs) Work



# The “Modus Ponens” Inference Rule

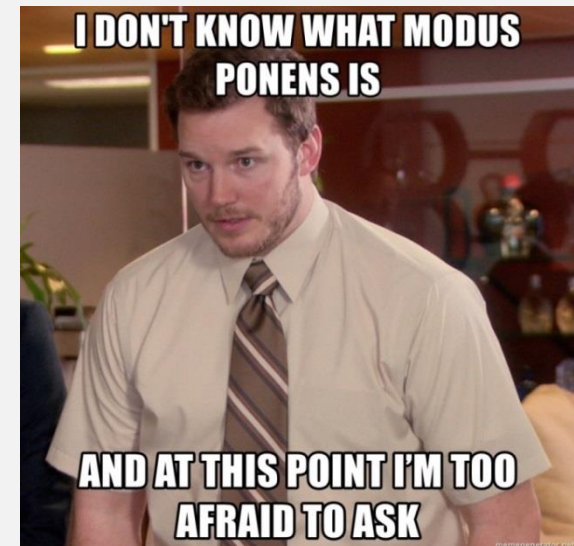
(Precisely Fitting Blocks Together)

**Premises** (if we can show these statements are true)

- If  $P$  then  $Q$
- $P$  is TRUE

**Conclusion** (then we can say that this is also true)

- $Q$  must also be TRUE





# Deductive Proof Example

Prove: fn f never throws ZeroDivisionError

```
def f(x):    "test expr"  
    if (x > 0) | (x < 0) | (x == 0):  
        return x + 1 "first branch"  
    else:  
        return 1 / 0 "second branch"
```

Proof:

Prior steps are already-proved, can be used to prove later steps!

## Statements / Justifications Table

### Statements

1. If running "test expr" is True, then "first branch" runs
2. If running "test expr" is False, then "second branch" runs
3. running "test expr" is (always) True
- 4. "first branch" (always) runs

### Justifications

1. Rules of Python
2. Rules of Python
3. Definition of "numbers"
4. By steps 1, 3, and modus ponens

Modus Ponens  
If we can prove these:

- If P then Q
- P

Then we've proved:

- Q ←

7. fn f never throws ZeroDivisionError

# Deductive Proof Example

Prove: fn f never throws ZeroDivisionError

```
def f(x):  
    if (x > 0) | (x < 0) | (x == 0):  
        return x + 1 "first branch"  
    else:  
        return 1 / 0 "second branch"
```

Proof:

Statements / Justifications Table

## Statements

1. If running "test expr" is True, then "first branch" runs
2. If running "test expr" is False, then "second branch" runs
3. running "test expr" is (always) True
4. "first branch" (always) runs
5. "second branch" never runs
6. fn f never runs 1 / 0
- ➔ 7. fn f never throws ZeroDivisionError

## Justifications

1. Rules of Python
2. Rules of Python
3. Definition of "numbers"
4. By steps 1, 3, and modus ponens
5. By steps 1, 2, and ???
6. By step 5
7. By step 6 and ???

# What else can we prove about programs?

```
function check(n)
{ // check if the number n is a prime
  var factor; // if the checked number is not a prime, this is its first factor
  var c;
  factor = 0;
  // try to divide the checked number by all numbers till its square root
  for (c=2; (c <= Math.sqrt(n)); c++)
  {
    if (n%c == 0) // is n divisible by c ?
      { factor = c; break }
  }
  return (factor);
} // end of check function

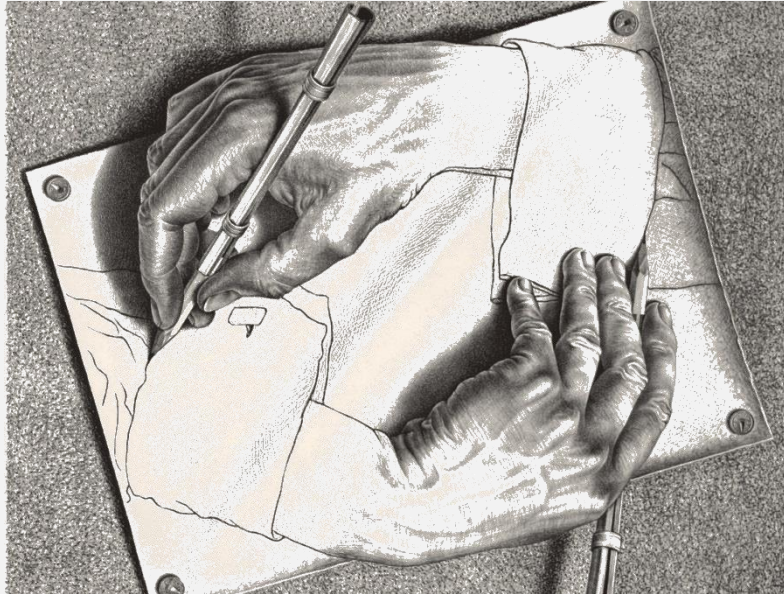
function communicate()
{ // communicate with the user
  var i; // i is the checked number
  var factor; // if the checked number is not a prime, this is its first factor
  i = document.primetest.number.value; // get the checked number
  // is it a valid input?
  if ((i != N(i)) || (i <= 0) || (Math.floor(i) != i))
    { alert ("The checked object should be a whole positive number"); }
  else
  {
    factor = check (i);
    if (factor == 0)
      { alert (i + " is a prime number"); }
    else
      { alert (i + " is not a prime number. i =" + factor + "X" + i/factor) }
  }
} // end of communicate function
```

## RANSOMWARE ATTACK



Predict result without running a program?

# Can we make predictions about computation?



It's tricky: **Trying to predict computation requires computation!**

# Can we make predictions about computation?

- The **Halting Lemma** says:



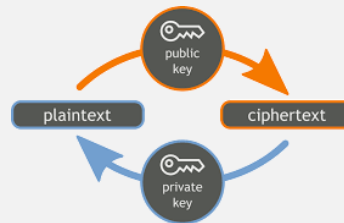
- And **Rice's Theorem** says:

- “all non-trivial, semantic properties of programs are undecidable”

# Knowing What Computers Can't Do is Still Useful!

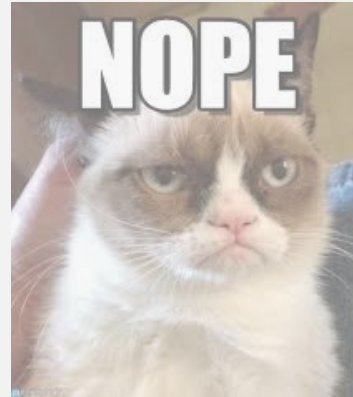
In Cryptography:

- Perfect secrecy is impossible in practice
- But with slightly imperfect secrecy (i.e., a computationally bounded adversary) we get:



# Can we make predictions about computation?

- The **Halting Lemma** says:



- And **Rice's Theorem** says:

- “all non-trivial, semantic properties of programs are undecidable”

Actually:

- it depends on the computation model!



# Predicting What Some Programs Will Do ...

microsoft.com/en-us/research/project/slam/

SLAM is a project for checking that software satisfies critical behavioral properties of the interfaces it uses and to aid software engineers in designing interfaces and software that ensure reliable and correct functioning. Static Driver Verifier is a tool in the Windows Driver Development Kit that uses the SLAM verification engine.

*"Things like even software verification, this has been the Holy Grail of computer science for many decades but now in some very key areas, for example, driver verification we're building tools that can do actual proof about the software and how it works in order to guarantee the reliability." Bill Gates, April 18, 2002. [Keynote address at WinHec 2002](#)*

SLAM  
Microsoft Research  
{ocs, end()}node}

Predicting things about programs ... is the Holy grail of CS!

Static Driver Verifier Research Platform README

## Overview of Static Driver Verifier Research Platform

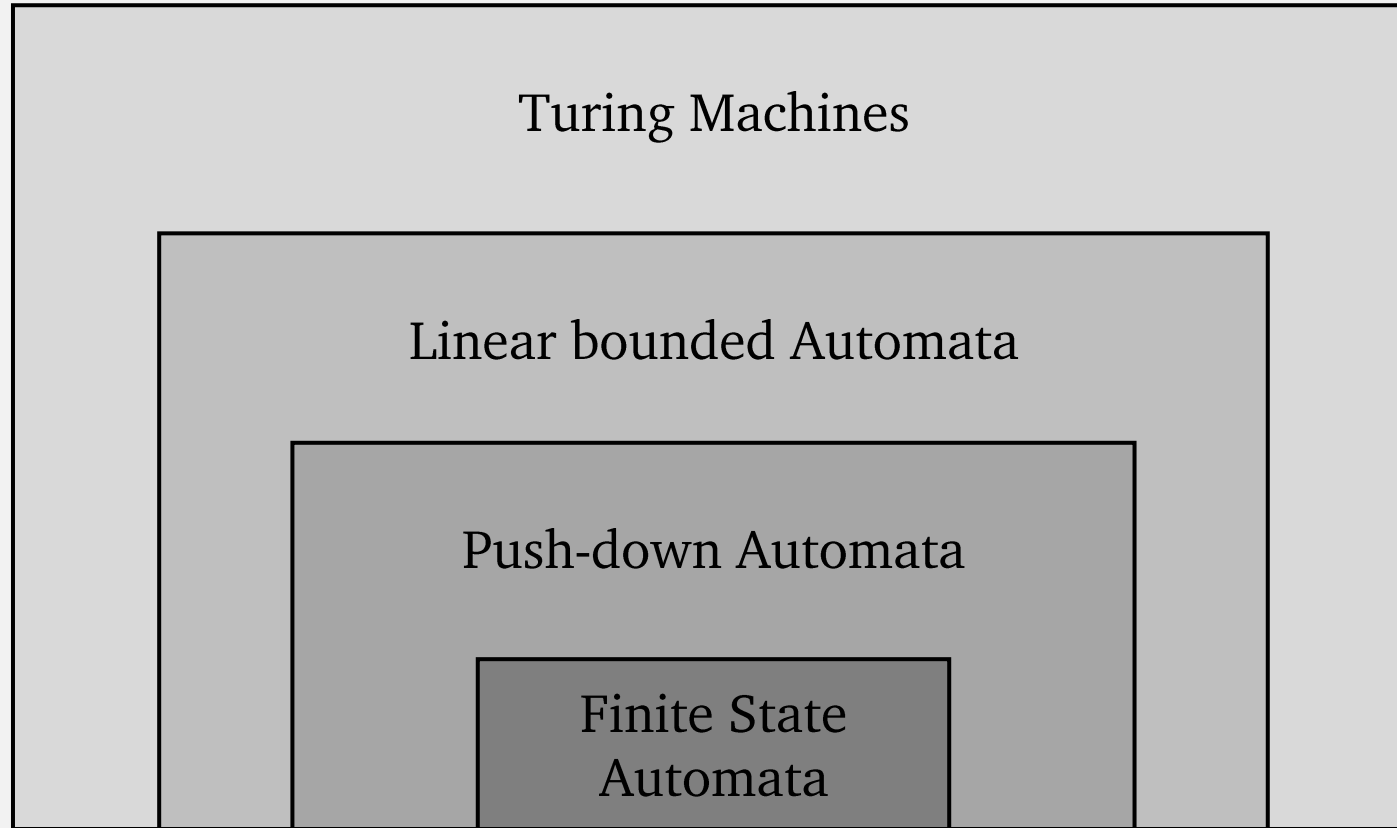
Static Driver Verifier (SDV) is a compile-time static verification tool, included in the Windows Driver Kit (WDK). The SDV Research Platform (SDVRP) is an extension to SDV that allows you to adapt SDV to:

- Support additional frameworks (or APIs) and write custom SLIC rules for this framework.
- Experiment with the model checking step.

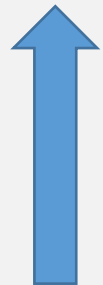




# CS 420 Proofs About Computational Models

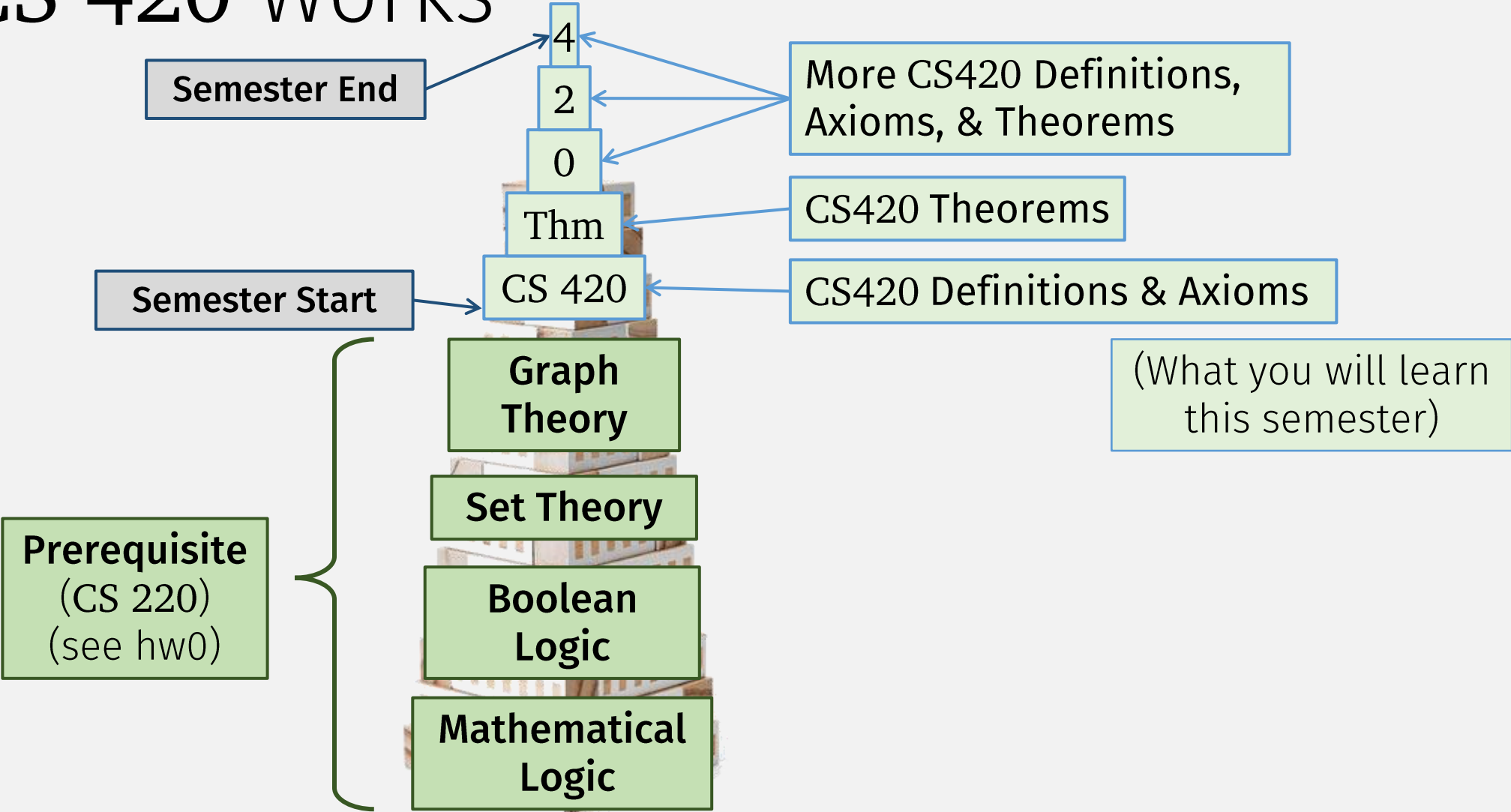


More powerful  
More complex  
Less restricted



In this class, we will prove things about our simple computational models

# How CS 420 Works



# A Word of Advice

Important:  
**Do not fall behind**  
in this course



To prove a (new) theorem ...

... need to know all axioms,  
definitions, and (previous)  
theorems below it

# Another Word of Advice

HW 1, Problem 1

Prove that  $ABC = XYZ$



How can I help you today?

Message ChatGPT... Prove that  $ABC = XYZ$

A Not-From-CS42 -  
Spring2024 Theorem



**“Blocks” from outside the course won’t work in the proof**

Remember:

**Preciseness in proofs** (just like in programming) **is critical**  
(Proofs must connect facts from this course exactly)



... can be used to **prove** (new) **theorems** in this course

Only axioms, definitions, and **theorems** from this course...

HW problems are *graded* on precise steps in the proof, not on the final theorem itself!

# Textbooks

- Sipser. *Intro to Theory of Computation*, 3<sup>rd</sup> ed.
- Hopcroft, Motwani, Ullman. *Intro to Automata Theory, Languages, and Computation*, 3<sup>rd</sup> ed.

**Strongly Recommended** (but not required)

- **Slides** (posted) and **lecture** should be **self-contained**,
- **BUT, Students who do well read the book**

All course info available on web site:  
<https://www.cs.umb.edu/~stchang/cs420/s24>

# How to Do Well in this Course

- Learn the “building blocks”
  - i.e., axioms, definitions, and theorems
- To solve a problem (prove a new theorem) ...  
... think about how to (precisely) combine existing “blocks”
- HW problems graded on steps to the answer (not final theorem)
- Don't Fall Behind!
  - Start HW Early (HW 0 due Monday 1/22 12pm EST noon)
- Participate and Engage
  - Lecture
  - Office Hours
  - Message Boards (piazza)

# Grading

- **HW: 80%**
  - Weekly: In / Out Monday
  - Approx. 12 assignments
  - Lowest grade dropped
- **Participation: 20%**
  - Lecture participation, in-class work, office hours, piazza
- **No exams**
- **A range: 90-100**
- **B range: 80-90**
- **C range: 70-80**
- **D range: 60-70**
- **F: < 60**

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# Late HW

- Is bad ... try not to do it please
  - Grades get delayed
  - Can't discuss solutions
  - You fall behind!
- Late Policy: **3 late days** to use during the semester



# HW Collaboration Policy

## Allowed

- Discussing HW with classmates (but must cite)
- Using other resources to learn, e.g., youtube, other textbooks, ...
- Writing up answers on your own, from scratch, in your own words

## Not Allowed

- Submitting someone else's answer
- Submitting someone else's answer with:
  - variables changed,
  - thesaurus words,
  - or sentences rearranged ...
- Using sites like Chegg, CourseHero, Bartleby, Study, ChatGPT, etc.
- Using theorems or definitions not from this course

# Honesty Policy

- 1<sup>st</sup> offense: zero on problem
- 2<sup>nd</sup> offense: zero on hw, reported to school
- 3<sup>rd</sup> offense+: F for course

## Regret policy

- If you self-report an honesty violation, you'll only receive a zero on the problem and we move on.

# All Up to Date Course Info

Survey, Schedule, Office Hours, HWs, ...

See course website:

<https://www.cs.umb.edu/~stchang/cs420/s24/>

**hw0 (pre-req quiz)**  
(see gradescope)