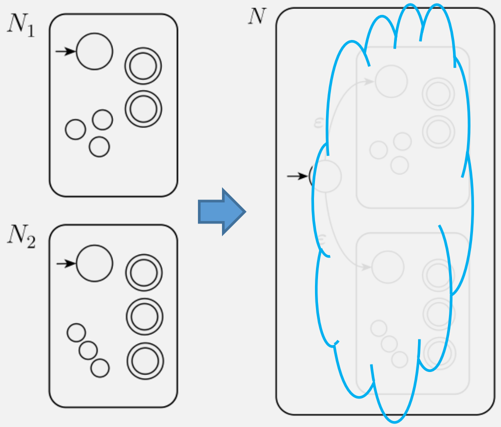
CS420

Combining DFAs and Closed Operations

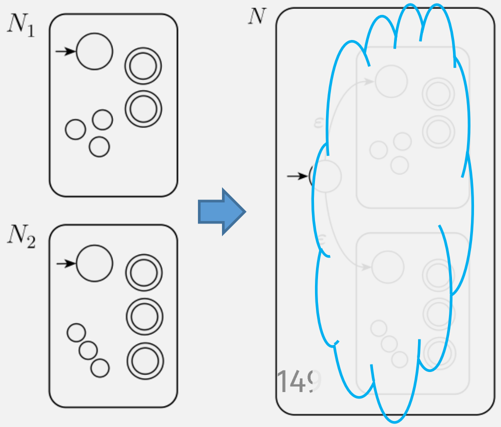
Monday, February 12, 2024

UMass Boston Computer Science



Announcements

- HW 1 in
  - ~~Due Mon 2/12 12pm~~
- HW 2 out
  - Due Mon 2/19 12pm
- Check previous Piazza posts before posting!



Previously

Languages Are Computation Models

- The **language** of a **machine** = set of strings that it **accepts**

M ***accepts*** w if $\hat{\delta}(q_0, w) \in F$

  - E.g., a DFA $M = (Q, \Sigma, \delta, q_0, F)$ **recognizes** language A: if $A = \{w|\ M \text{ accepts } w\}$

- A **computation model** = set of machines it defines
  - E.g., **all possible DFAs** are a **computation model**

DEFINITION

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, wher

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta \colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

Thus: a **computation model** equivalently = a set of languages

= set of set of strings

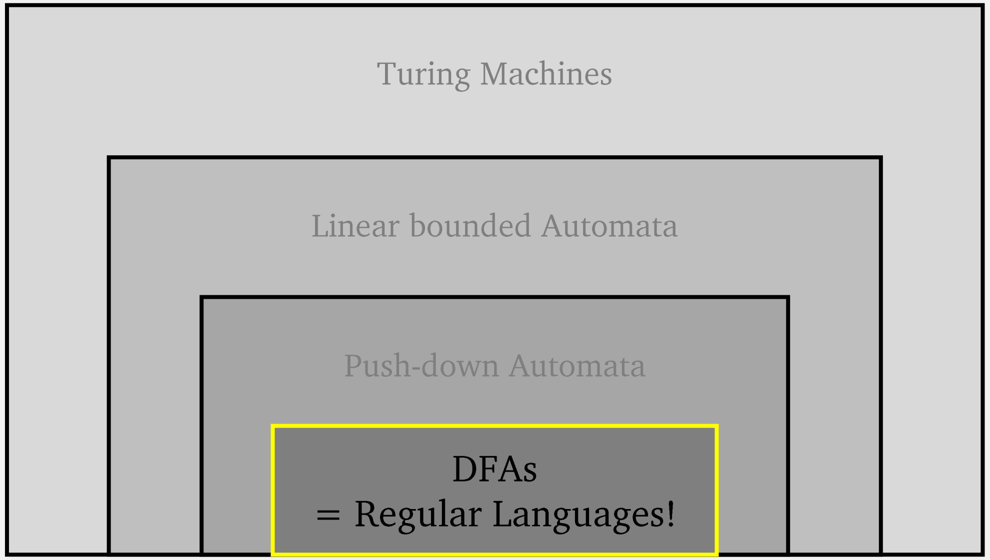
This class is really about studying **sets of languages!**

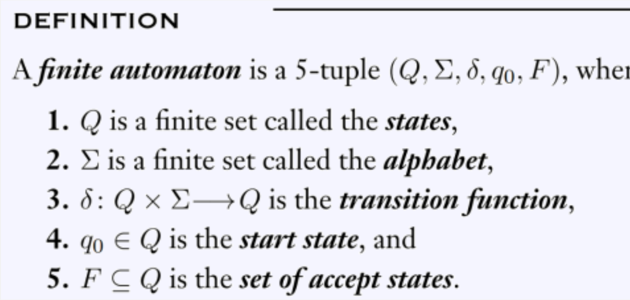
Previously

Languages Are Computation Models

- first **set of languages** we will study: **regular languages**

If a **DFA recognizes** a **language** L, then L is a **regular language**

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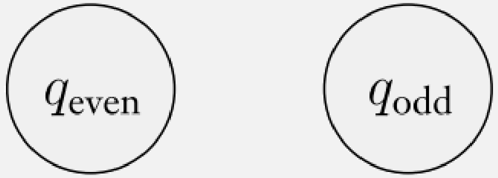
Previously

Is it regular?: strings with odd # **1**s

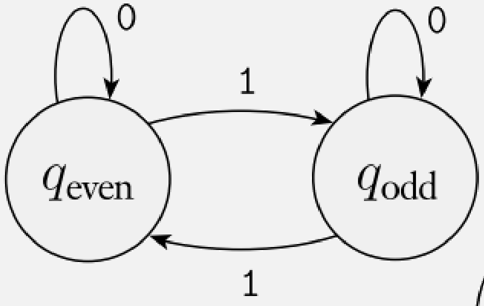
(Part of Proof requires)
Creating DFA:

- **States:**
  - 2 states:
    - seen even 1s so far
    - seen odds 1s so far

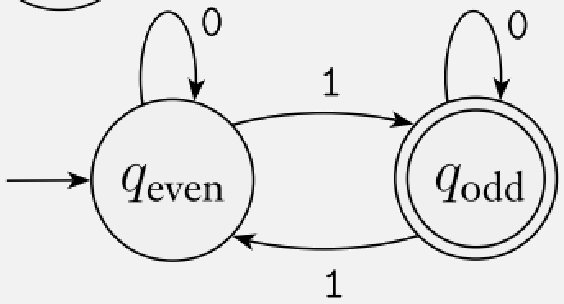


- **Alphabet:** `0` and `1`

- **Transitions:**



- **Start** / **Accept** states:



So a DFA's computation recognizes simple string patterns?

Yes!

Have you ever used a **programming language** (feature) for writing string matching computation?

Regular Expressions!
(stay tuned!)

Combining DFAs?

Password Requirements

- Passwords must have a minimum length of ten (10) characters - but more is better! ← DFA
- Passwords **must include at least 3** different types of characters:
  - upper-case letters (A-Z) ← DFA
  - lower-case letters (a-z) ← DFA
  - symbols or special characters (%, &, *, $, etc.) ← DFA
  - numbers (0-9) ← DFA
- Passwords cannot contain all or part of your email address ← DFA
- Passwords cannot be re-used ← DFA

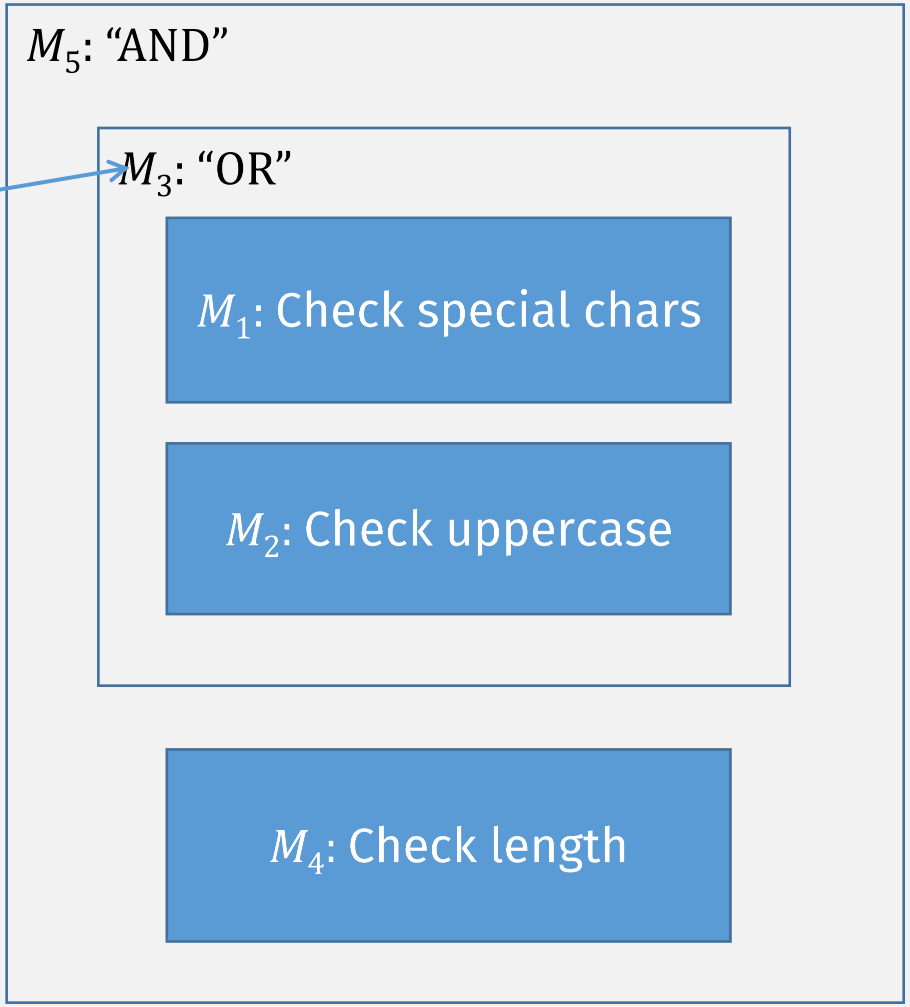
To match <u>all</u> requirements, <u>combine</u> smaller DFAs into one big DFA?

umb.edu/it/software-systems/password/

(We do this with programs all the time)

Password Checker DFAs

To combine more than once, this must be a DFA



Want to be able to easily combine DFAs, i.e., composability

We want these operations:

"OR" : DFA × DFA → DFA

"AND" : DFA × DFA → DFA

To combine more than once, operations must be **closed**!

"Closed" Operations

- Set of Natural numbers = $\{0, 1, 2, ...\}$
  - Closed under addition:
    - if x and y are Natural numbers,
    - then $z = x + y$ is a Natural number
  - Closed under multiplication?
    - **yes**
  - Closed under subtraction?
    - **no**
- Integers = $\{..., -2, -1, 0, 1, 2, ...\}$
  - Closed under addition and multiplication
  - Closed under subtraction?
    - **yes**
  - Closed under division?
    - **no**
- Rational numbers = $\{x \mid x = y/z, y$ and z are Integers$\}$
  - Closed under division?
    - **No?**
    - **Yes if z !=0**

A set is **closed** under an operation if: the result of applying the operation to members of the set is in the same set

i.e., input set(s) = output set

We Want "Closed" Ops For Regular Langs!

- Set of Regular Languages = $\{L_1, L_2, \ldots\}$
  - Closed under …?
    - OR (union)
    - AND (intersection)
    - …

A set is **closed** under an operation if:
the result of applying the operation to
members of the set is in the same set

i.e., input set(s) = output set

Why Care About Closed Ops on Reg Langs?

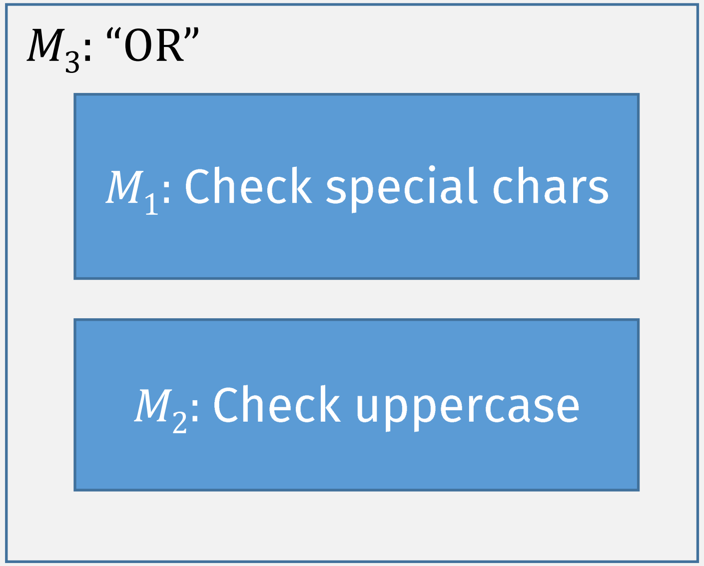
- Closed operations for regular langs preserve “regularness”
- I.e., **it preserves the same computation model!**
- Allows “combining” smaller “regular” computations to get bigger ones:

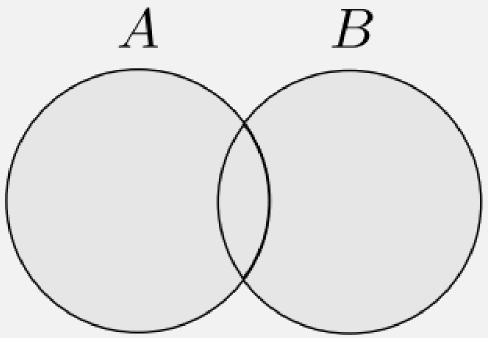
For Example:

OR: Regular Lang × Regular Lang → Regular Lang

- So this semester, **we will look for operations that are closed!**

Password Checker: “**OR**” = “Union”





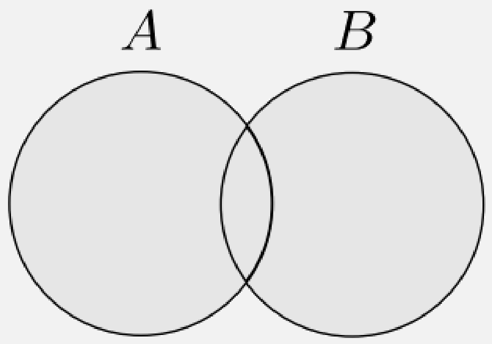
Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Union of Languages

Let the alphabet Σ be the standard 26 letters $\{\texttt{a}, \texttt{b}, \ldots, \texttt{z}\}$.

If $A = \{\texttt{fort}, \texttt{south}\}$ $B = \{\texttt{point}, \texttt{boston}\}$

$$A \cup B = \{\texttt{fort}, \texttt{south}, \texttt{point}, \texttt{boston}\}$$



Is Union Closed For Regular Langs?

In this course, **we are interested in closed operations for a** **<u>set of languages</u>** (here the **set of regular languages**)

(*In general, a* **set** is **closed** under an operation if **applying the operation** to **members of the set** produces a **result in the same set**)

The class of regular languages is closed under the union operation.

Want to prove this statement

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Or this (same) statement

Is Union Closed For Regular Langs?

(*In general*, a **set** is **closed** under an operation if applying the operation to members of the set produces a result in the same set)

THEOREM

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Want to prove this statement

Or this (same) statement

A member of the set of regular languages is …

… a regular language, which itself is a set (of strings) …

… so the **operations** we're interested in are **set operations**

Is Union Closed For Regular Langs?

THEOREM

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Want to prove this statement

Or this (same) statement

Flashback: Mathematical Statements: **IF-THEN**

Using:

- If we know: $P \to Q$ is TRUE,
  what do we know about P and Q individually?
  - Either P is FALSE (not too useful, **can't prove anything about** Q), or
  - If P is TRUE, then Q is TRUE (**modus ponens**)

Proving:

p	q	$p \to q$
True	True	True
True	False	False
False	True	True
False	False	True

Flashback: Mathematical Statements: **IF-THEN**

THEOREM

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

t Q), or

• If P is TRUE, then Q is TRUE (**modus ponens**)

Would have to prove there are no regular languages (impossible)

Proving:

- To prove: $P \rightarrow Q$ is TRUE:
  - Prove P is FALSE (usually hard or impossible)
  - Assume P is TRUE, then prove Q is TRUE

p	q	$p \rightarrow q$
True	True	True
True	False	False
False	True	True
False	False	True

Is Union Closed For Regular Langs?

Statements

Do we know anything about A_1 and A_2?

1. A_1 and A_2 are regular languages
2. A DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes A_1
3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2
4. Construct DFA $M = (Q, \Sigma, \delta, q_0, F)$ (todo)
5. M recognizes $A_1 \cup A_2$
6. $A_1 \cup A_2$ is a regular language
7. The class of regular languages is closed under the union operation.

   In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

???

How to create this? Don't know what A_1 and A_2 are!

Justifications

1. Assumption
2. Def of Regular Language
3. Def of Regular Language
4. Def of DFA
5. See examples
6. Def of Regular Language
7. From stmt #1 and #6

Wait! If A Then B =?= If B Then A

If a **DFA** recognizes a language L, then L is a **regular language**

1. A_1 and A_2 are regular languages
2. A DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes A_1 — 2. Def of Regular Language
3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2 — 3. Def of Regular Language

If L is a **regular language**, then a **DFA** recognizes L ???

Equivalence of Conditional Statements

- Yes or No? "If X then Y" is equivalent to:
    - "If Y then X" (**converse**)
        - No!

If Regular, Then DFA?

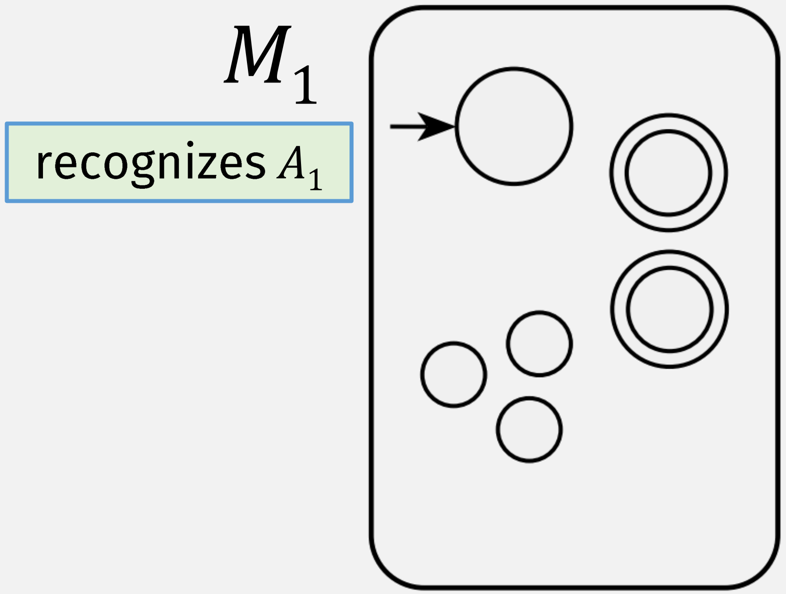
If a **DFA recognizes** a language L, then L is a **regular language**

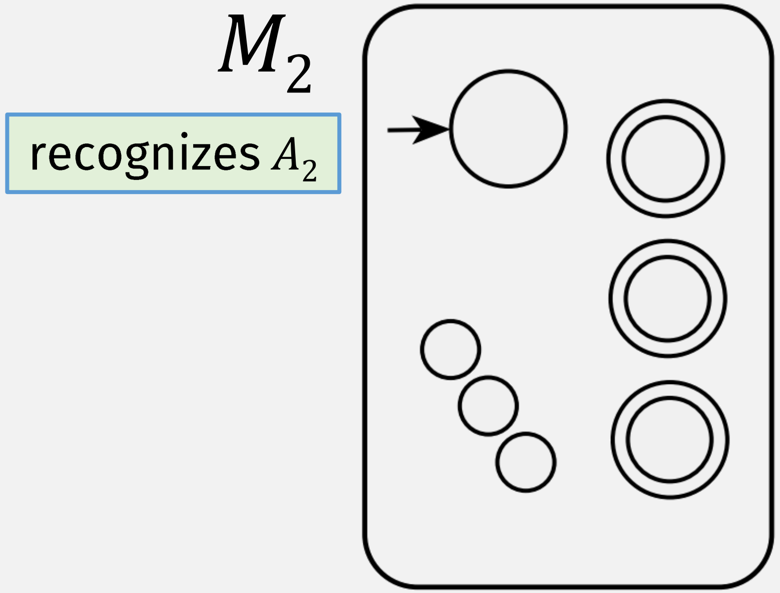
- Prove: If L is a **regular language**, then a **DFA recognizes** L
- Proof (Sketch)

  Case analysis:
  - **Look at all** If-then **statements** of the form:
    - "If … language L, then L is a **regular language**"
  - (At least one is true!)
  - **Figure out which one(s)** led to conclusion:
    - "L is a **regular language**"
  - (There's only 1!)

  - So it must be that:

If L is a **regular language**, then a **DFA recognizes** L





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4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

Regular language A_1
Regular language A_2

Even if we don't know what these languages are, we still know…

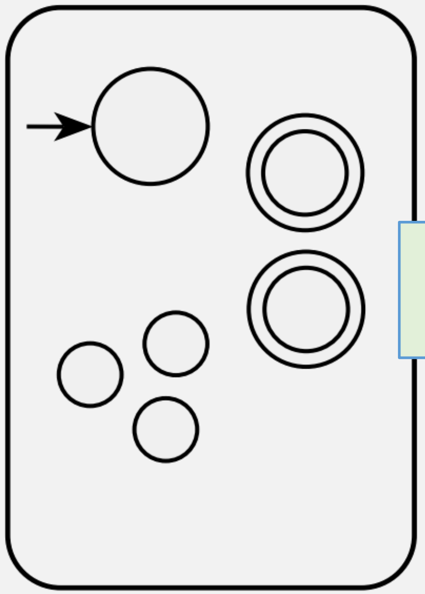
$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,

If L is a **regular language**, then a **DFA** recognizes L

Union

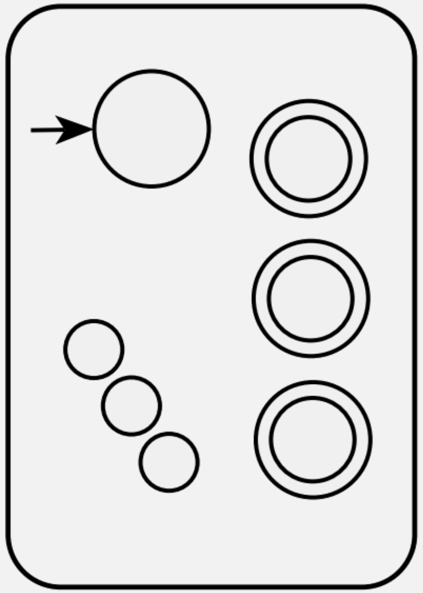
M_1

recognizes A_1



M_2

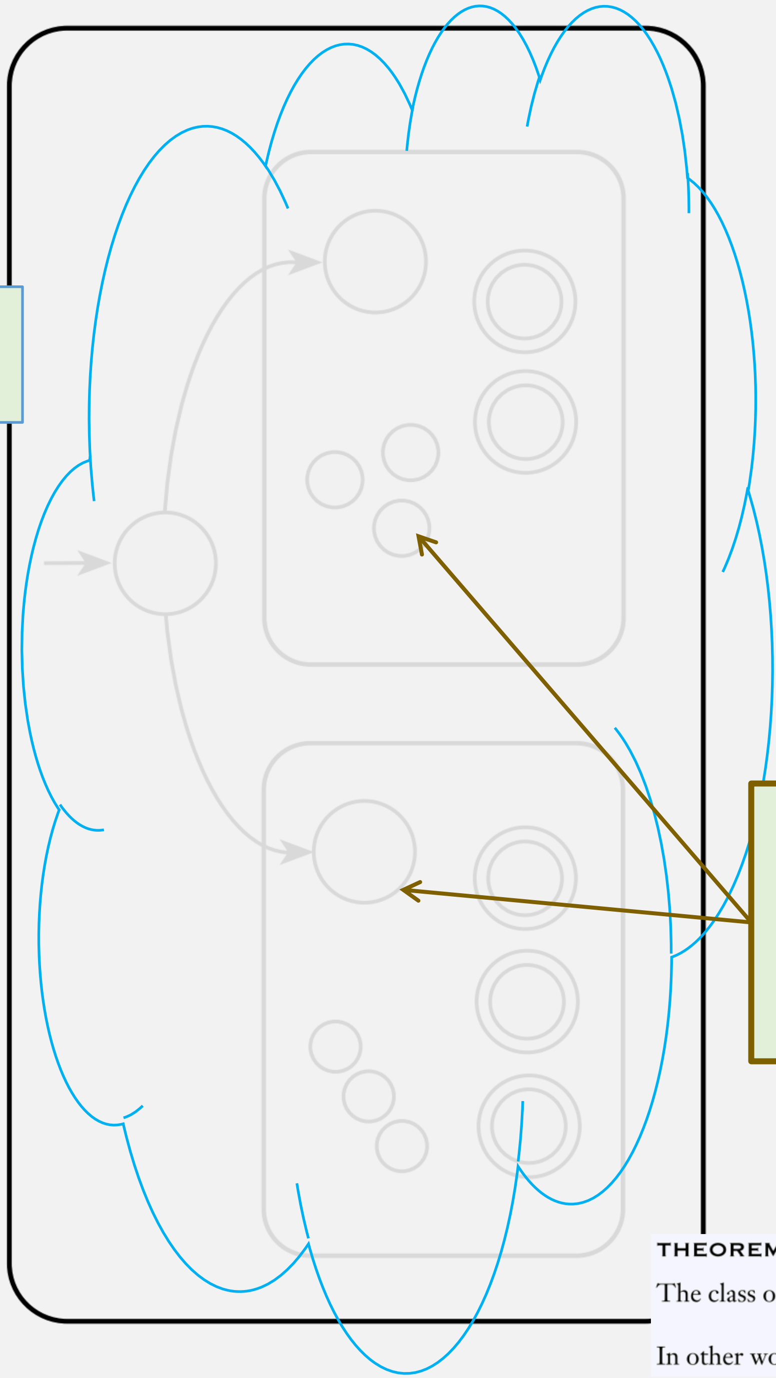
recognizes A_2



<u>Want:</u> M

Recognizes $A_1 \cup A_2$

(to prove $A_1 \cup A_2$ is regular)



<u>Rough sketch Idea:</u>
M is a <u>combination</u> of M_1 and M_2 that: checks whether its input is accepted by <u>either</u> M_1 or M_2

But, a DFA can only read its input <u>once</u>!

Need to somehow simulate "being in" <u>both</u> an M_1 and M_2 state simultaneously

THEOREM

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Previously

Is Union Closed For Regular Langs?

Statements

1. A_1 and A_2 are regular languages
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3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2
4. ??? Construct DFA $M = (Q, \Sigma, \delta, q_0, F)$ (todo)
5. M recognizes $A_1 \cup A_2$
6. $A_1 \cup A_2$ is a regular language
7. The class of regular languages is closed under the union operation.

   In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

How to create this? Don't know what A_1 and A_2 are!

Justifications

1. Assumption
2. Def of Regular Language
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Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,

Want: M that can simultaneously "be in" both an M_1 and M_2 state

- Construct: $M = (Q, \Sigma, \delta, q_0, F)$, using M_1 and M_2, that recognizes $A_1 \cup A_2$
- **states of M:** $Q = \{(r_1, r_2) | \, r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$
  This set is the ***Cartesian product*** of sets Q_1 and Q_2

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

A state of M is a pair:
- the first part is a state of M_1 and
- the second part is a state of M_2

So the states of M is all possible combinations of the states of M_1 and M_2

Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
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$a) = (\delta_1(r_1, a), \delta_2(r_2, a))$

A step in M is both:
- a step in M_1, and
- a step in M_2

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- Construct: $M = (Q, \Sigma, \delta, q_0, F)$, using M_1 and M_2, that recognizes $A_1 \cup A_2$
- states of M: $Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$
  This set is the ***Cartesian product*** of sets Q_1 and Q_2
- M transition fn: $\delta\big((r_1, r_2), a\big) = \big(\delta_1(r_1, a), \delta_2(r_2, a)\big)$
- M start state: (q_1, q_2)

Start state of M is both start states of M_1 and M_2

Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,
- Construct: $M = (Q, \Sigma, \delta, q_0, F)$, using M_1 and M_2, that recognizes $A_1 \cup A_2$
- states of M: $Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$
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- M transition fn: $\delta\big((r_1, r_2), a\big) = \big(\delta_1(r_1, a), \delta_2(r_2, a)\big)$
- M start state: (q_1, q_2)

Accept if <u>either</u> M_1 or M_2 accept

Remember:
Accept states must be subset of Q

- M accept states: $F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\}$

Q.E.D.? ■

Previously

Is Union Closed For Regular Langs?

Statements	Justifications
1. A_1 and A_2 are regular languages	1. Assumption
2. A DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes A_1	2. Def of Regular Language
3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2	3. Def of Regular Language
??? 4. Construct DFA $M = (Q, \Sigma, \delta, q_0, F)$ ☑	4. Def of DFA
5. M recognizes $A_1 \cup A_2$	5. See examples
6. $A_1 \cup A_2$ is a regular language	6. Def of Regular Language
7. The class of regular languages is closed under the union operation. In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.	7. From stmt #1 and #6

How to create this? Don't know what A_1 and A_2 are!

"Prove" that DFA recognizes a language

Let $s_1 \in A_1$ and $s_2 \in A_2$
Let $s_3 \notin A_1$ and $s_4 \notin A_2$

Be careful when choosing examples!

String	In lang $A_1 \cup A_2$?	Accepted by M ?
s_1	Yes	
s_2	Yes	
s_3	???	
s_4	???	

Don't know A_1 and A_2 exactly …

… but we know …

… they are **sets of strings**!

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,

constructed $M = (Q, \Sigma, \delta, q_0, F)$ recognizes $A_1 \cup A_2$?

In this class, **a table like this is sufficient to "prove" that a DFA recognizes a language**

"Prove" that DFA recognizes a language

Let $s_1 \in A_1$ and $s_2 \in A_2$

~~Let $s_3 \notin A_1$ and $s_4 \notin A_2$~~

Let $s_5 \notin A_1$ and $\notin A_2$

String	In lang $A_1 \cup A_2$?	Accepted by M ?
s_1	Yes	
s_2	Yes	
~~s_3~~	~~???~~	
~~s_4~~	~~???~~	
s_5	No	

$$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1), \text{ recognize } A_1,$$
$$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2), \text{ recognize } A_2,$$

constructed $M = (Q, \Sigma, \delta, q_0, F)$ recognizes $A_1 \cup A_2$?

Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,
- Construct: $M = (Q, \Sigma, \delta, q_0, F)$, using M_1 and M_2, that recognizes $A_1 \cup A_2$
- states of M: $Q = \{(r_1, r_2) | \; r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$
  This set is the ***Cartesian product*** of sets Q_1 and Q_2
- M transition fn: $\delta\big((r_1, r_2), a\big) = \big(\delta_1(r_1, a), \delta_2(r_2, a)\big)$
- M start state: (q_1, q_2)

Accept if either M_1 or M_2 accept

- M accept states: $F = \{(r_1, r_2) | \; r_1 \in F_1 \text{ or } r_2 \in F_2\}$

“Prove” that DFA recognizes a language

Let $s_1 \in A_1$ and $s_2 \in A_2$

Let $s_5 \notin A_1$ and $\notin A_2$

String	In lang $A_1 \cup A_2$?	Accepted by M ?
s_1	Yes	Accept
s_2	Yes	Accept
~~s_3~~	~~???~~	???
~~s_4~~	~~???~~	???
s_5	No	Reject

$$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1), \text{ recognize } A_1,$$
$$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2), \text{ recognize } A_2,$$

constructed $M = (Q, \Sigma, \delta, q_0, F)$

Accept if <u>either</u> M_1 or M_2 accept

Previously

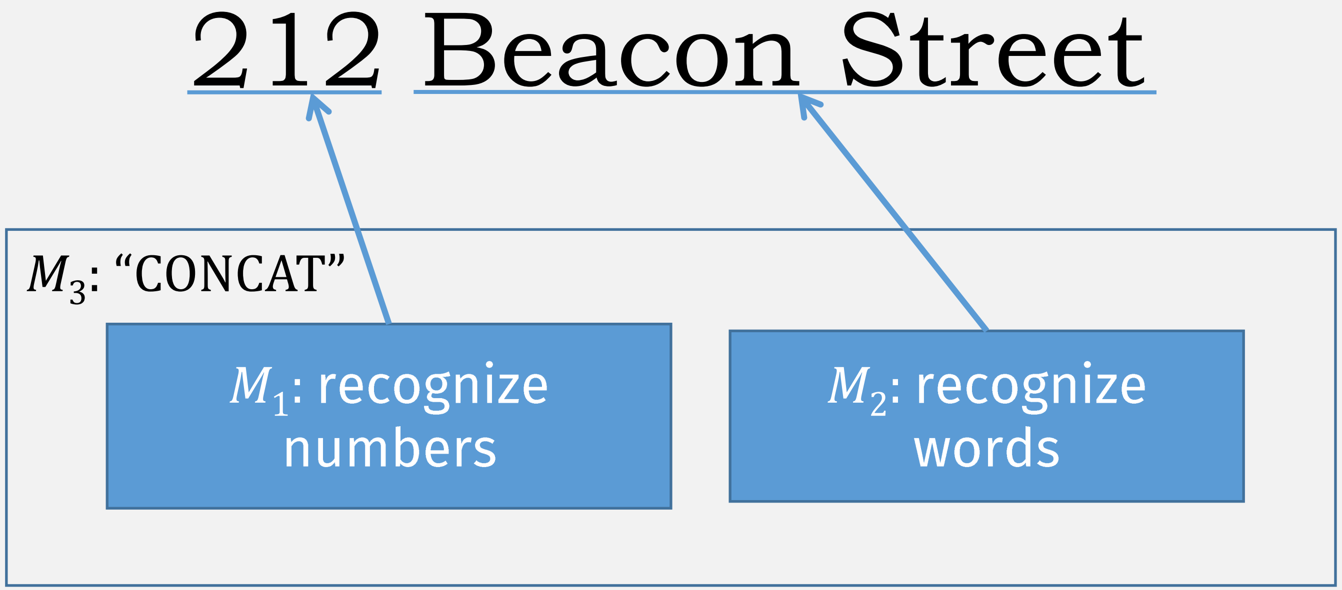
Is Union Closed For Regular Langs?

Statements	Justifications
1. A_1 and A_2 are regular languages	1. Assumption
2. A DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes A_1	2. Def of Regular Language
3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2	3. Def of Regular Language
4. Construct DFA $M = (Q, \Sigma, \delta, q_0, F)$ ☑	4. Def of DFA
5. M recognizes $A_1 \cup A_2$	5. See examples ☑
6. $A_1 \cup A_2$ is a regular language	6. Def of Regular Language
7. The class of regular languages is closed under the union operation. In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.	7. From stmt #1 and #6

Q.E.D. ■

Another operation: Concatenation

Example: **Recognizing street addresses**



Concatenation: $A \circ B = \{xy | \, x \in A \text{ and } y \in B\}$

Concatenation of Languages

Let the alphabet Σ be the standard 26 letters $\{\texttt{a}, \texttt{b}, \ldots, \texttt{z}\}$.

If $A = \{\texttt{fort}, \texttt{south}\}$ $B = \{\texttt{point}, \texttt{boston}\}$

$$A \circ B = \{\texttt{fortpoint}, \texttt{fortboston}, \texttt{southpoint}, \texttt{southboston}\}$$

Is Concatenation Closed?

THEOREM

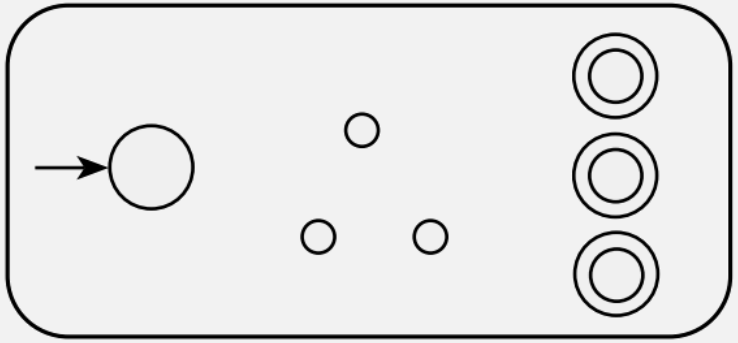
The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

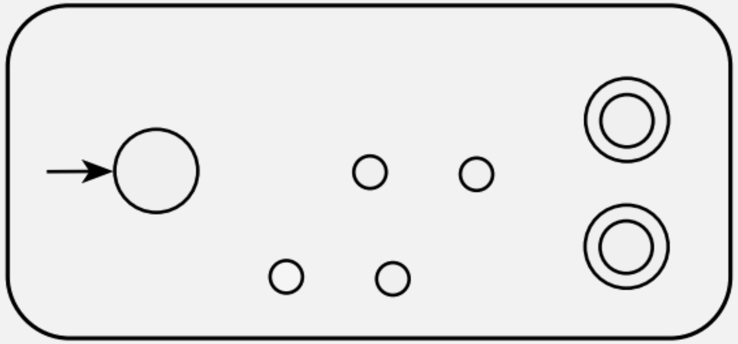
- **Construct a new machine M recognizing $A_1 \circ A_2$?** (like union)
  - Using **DFA M_1** (which recognizes A_1),
  - and **DFA M_2** (which recognizes A_2)

Concatentation

M_1



M_2

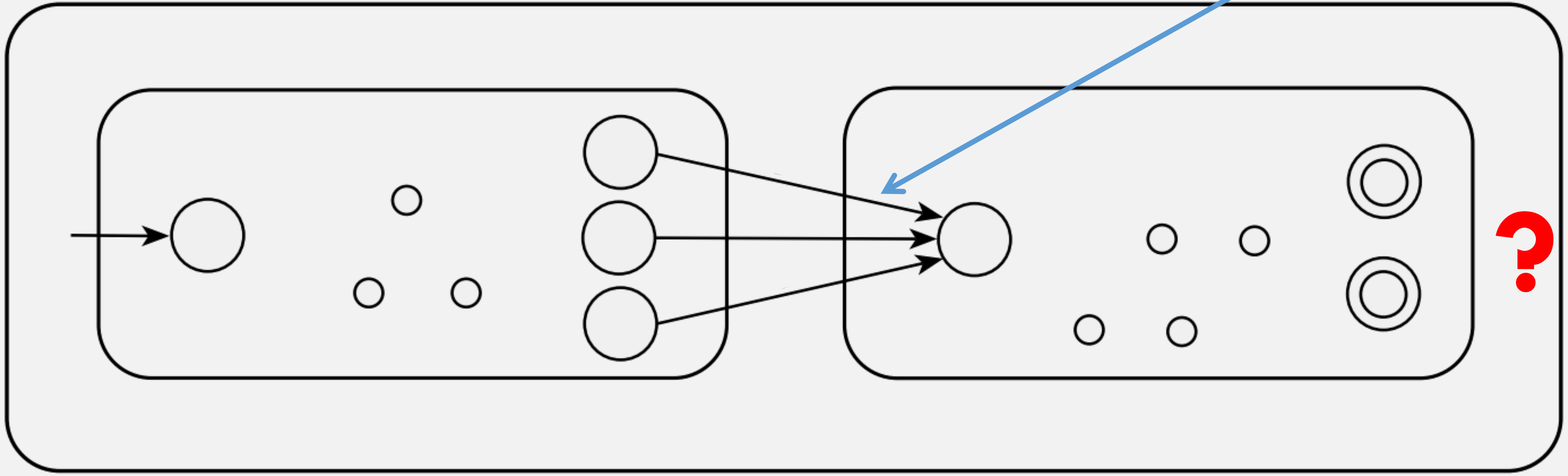


<u>PROBLEM</u>: Can only read input once, can't backtrack

Let M_1 recognize A_1, and M_2 recognize A_2.

<u>Want</u>: Construction of M to recognize $A_1 \circ A_2$

Need to switch machines at some point, but when?



???

Concatenation: $A \circ B = \{xy | x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 recognize language A = { `jen`, `jens` }
- and M_2 recognize language B = { `smith` }
- Want: Construct M to recognize $A \circ B$ = { `jensmith`, `jenssmith` }

- If M sees `jen` ...
- M must decide to <u>either</u>:

Concatenation: $A \circ B = \{xy | x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 recognize language A = { `jen`, `jens` }
- and M_2 recognize language B = { `smith` }
- Want: Construct M to recognize $A \circ B$ = { `jensmith`, `jenssmith` }

- If M sees `jen` ...
- M must decide to <u>either</u>:
  - stay in M_1 (correct, if full input is `jens``smith`)

Concatenation: $A \circ B = \{xy | x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 recognize language A = { `jen`, `jens` }
- and M_2 recognize language B = { `smith` }
- Want: Construct M to recognize $A \circ B$ = { `jensmith`, `jenssmith` }

- If M sees `jen` ...
- M must decide to either:
  - stay in M_1 (correct, if full input is `jenssmith`)
  - or switch to M_2 (correct, if full input is `jensmith`)
- But to recognize $A \circ B$, it needs to handle both cases!!
  - Without backtracking

A **DFA** can't do this!

Is Concatenation Closed?

FALSE?

THEOREM

The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

- Cannot **combine A_1 and A_2's machine because:**
  - **Need to switch from A_1 to A_2 at some point …**
  - **… but we don't know when!** (we can only read input once)
- **This requires a new kind of machine!**
- But **does this mean concatenation is not closed for regular langs?**