UMB CS 420

Non-CFLs

Wednesday, March 27, 2024

Announcements

- HW 6
  - Due Monday 4/1 12pm noon

Flashback: Pumping Lemma for Regular Langs

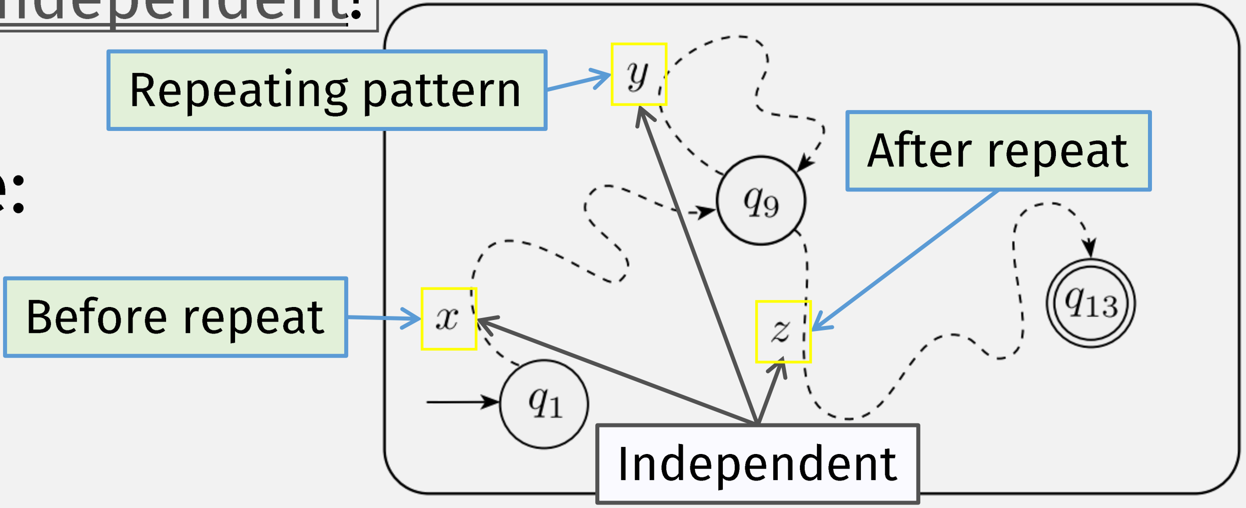
- **Pumping Lemma** describes how strings repeat

- **Regular language** strings repeat using **Kleene star** operation
  - Key: 3 substrings $x\,y\,z$ independent!

- A non-regular language:

$$\{0^n 1^n \mid n \geq 0\}$$

Kleene star can't express this pattern:
2nd part depends on (length of) 1st part



- Q: How do CFLs repeat?

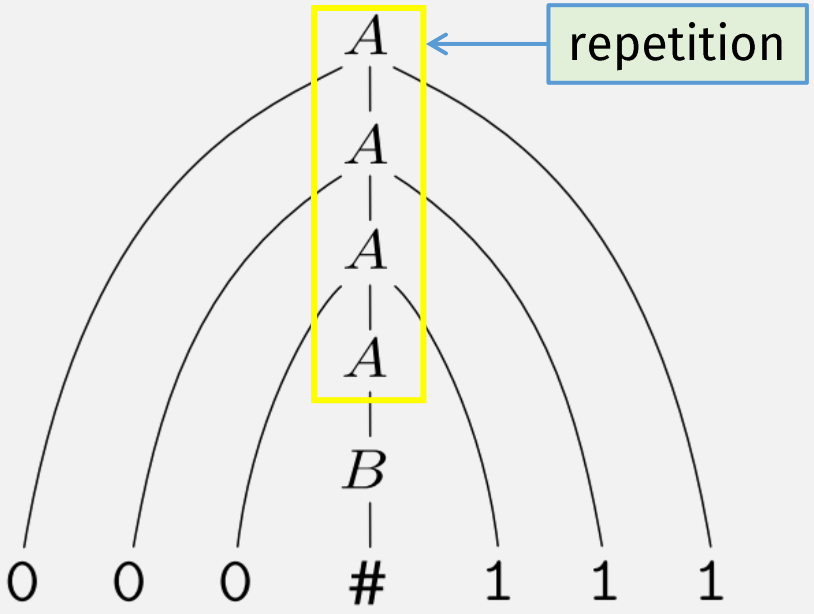
Repetition and Dependency in CFLs

Parts before/after repetition point <u>linked</u> (not independent)

Repetition

$$A \to 0A1$$
$$A \to B$$
$$B \to \#$$

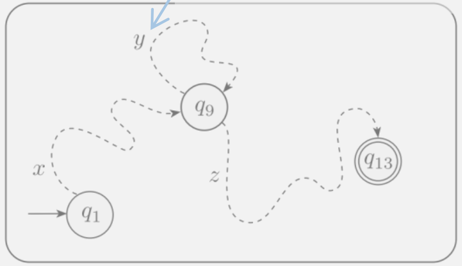
$$\{0^n\#1^n \mid n \geq 0\}$$



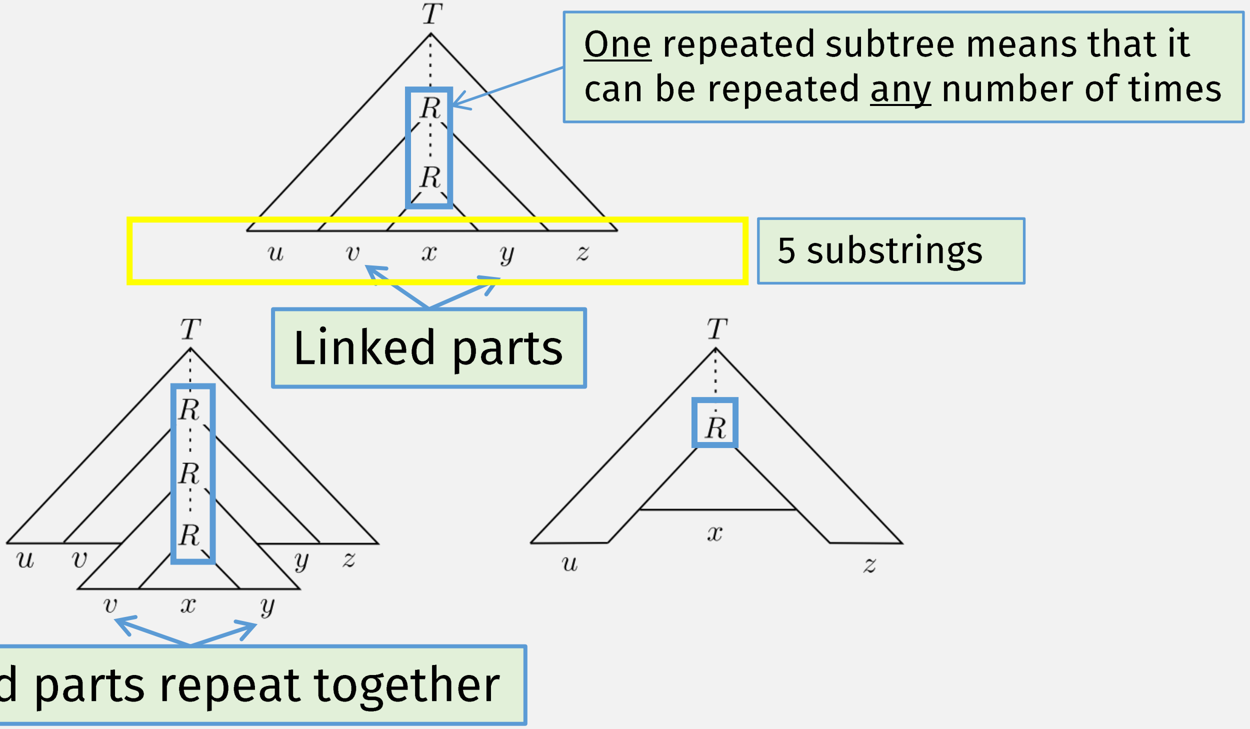
$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\#111$$

How Do Strings in CFLs Repeat?

NFA can take loop transition any number of times, to process repeated y in input



- Strings in regular languages <u>repeat states</u>
- Strings in CFLs <u>repeat subtrees</u> in the parse tree

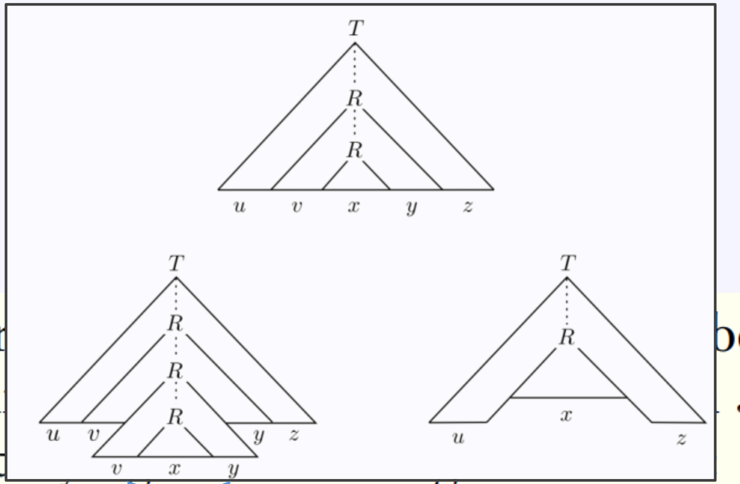


Pumping Lemma for CFLS

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

Two pumpable parts.
But they must be pumped together!

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.



Pumping lemma If A is a regular … ber p (the pumping length) where if s is any stri… s may be divided into three pieces, $s = xyz$, sat…

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

One pumpable part

Two pumpable parts, pumped together

Previously

A Non CFL example

$$\text{language } B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\} \text{ is not context free}$$

Intuition

- **Strings in CFLs** can have **two parts** that are **“pumped” together**
- **Language** B **requires three parts** to be **“pumped” together**
- So **it’s not a CFL!**

Proof?

Want to prove: $a^n b^n c^n$ **is not** a CFL

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

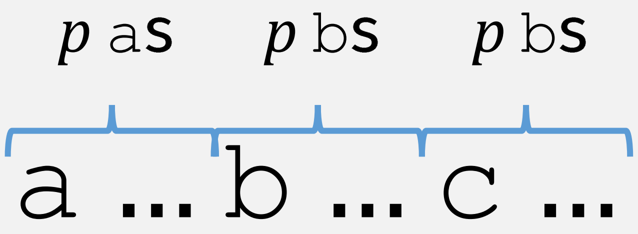
Reminder: CFL Pumping lemma says: all strings $a^n b^n c^n$ ≥ length p are splittable into *uvxyz* where *v* and *y* are pumpable

Proof (by contradiction):

Now we must find a contradiction …

- Assume: $a^n b^n c^n$ **is** a CFL
  - So it must satisfy the pumping lemma for CFLs
  - I.e., all strings ≥ length p are pumpable
- Counterexample = $a^p b^p c^p$

Contradiction if:
- A string in the language ☑
- ≥ length p ☑
- Is **not splittable** into *uvxyz* where *v* and *y* are pumpable ???



Want to prove: $a^n b^n c^n$ **is not** a CFL

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
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Possible Splits

Proof (by contradiction):

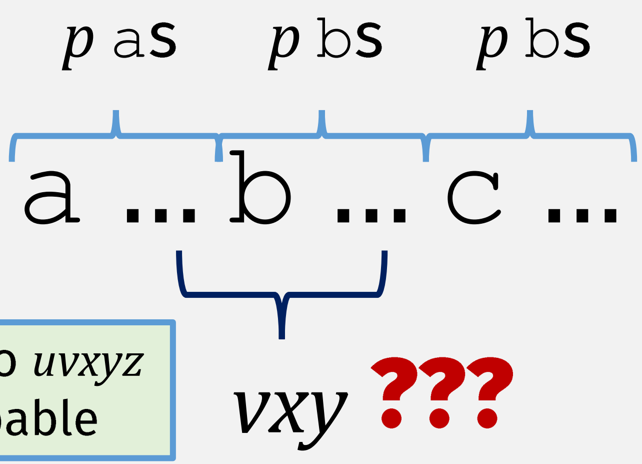
- Assume: $a^n b^n c^n$ **is** a CFL
  - So it must satisfy the pumping lemma for CFLs
  - I.e., all strings ≥ length p are pumpable
- Counterexample = $a^p b^p c^p$

Contradiction if:
- A string in the language
- ≥ length p
- Is **not splittable** into *uvxyz* where *v* and *y* are pumpable

contradiction

- Possible Splits (using condition # 3: $|vxy| \leq p$)
  - ☒ *vxy* is all as
  - ☒ *vxy* is all bs
  - ☒ *vxy* is all cs
  - ☒ *vxy* has as and bs
  - ☒ *vxy* has bs and cs
  - (*vxy* cannot have as, bs, and cs)

Not pumpable



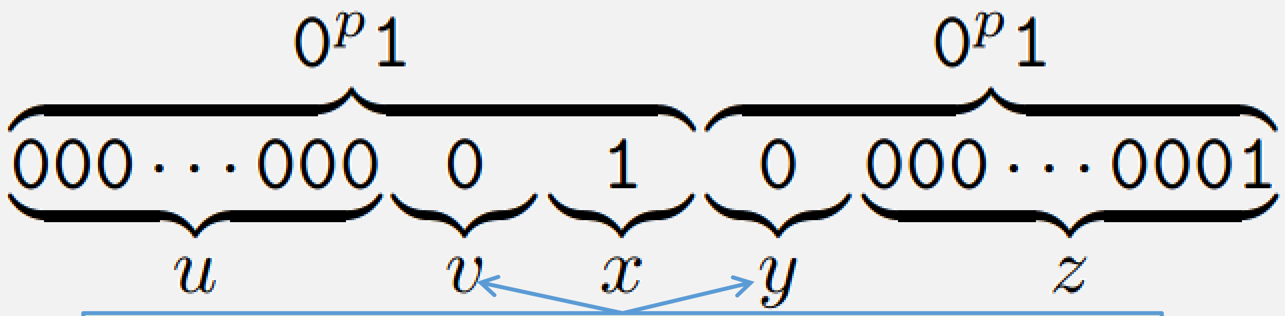
$a^p b^p c^p$ **cannot be split** into *uvxyz* where *v* and *y* are pumpable

So $a^n b^n c^n$ **is not a CFL**
(justification:
contrapositive of CFL pumping lemma)

Another Non-CFL $D = \{ww \mid w \in \{0,1\}^*\}$

Be careful when choosing **counterexample** *s*: 0^p10^p1

This *s* can be pumped according to CFL pumping lemma:



Pumping *v* and *y* (together) produces string still in *D!*

- **CFL Pumping Lemma conditions:**
  - ☑ **1.** for each $i \geq 0$, $uv^ixy^iz \in A$,
  - ☑ **2.** $|vy| > 0$, and
  - ☑ **3.** $|vxy| \leq p$.

So **this attempt** to **prove** that the **language** is **not a CFL failed**.
(It doesn't prove that the language is a CFL!)

Another Non-CFL $D = \{ww \mid w \in \{0,1\}^*\}$

- Need <u>another counterexample</u> string *s*:

If *vyx* is contained in first or second half, then any pumping will break the match

$0^p 1^p 0^p 1^p$

So *vyx* must straddle the middle

But any pumping still breaks the match because order is wrong

- CFL Pumping Lemma conditions:

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Now we have proven that this language is **not a CFL!**

A Practical Non-CFL

- **XML**
  - ELEMENT → <TAG>CONTENT</TAG>
  - Where TAG is any string

- XML also looks like this non-CFL: $D = \{ww|\ w \in \{0,1\}^*\}$

- This means XML is not context-free!
  - Note: HTML *is* context-free because ...
  - ... there are only a finite number of tags,
  - so they can be embedded into a finite number of rules.

In practice:
  - XML is parsed as a CFL, with a CFG
  - Then matching tags checked in a 2nd pass with a more powerful machine ...

Next: A More Powerful Machine …

M_1 accepts its input if it is in language: $B = \{w\#w \mid w \in \{0,1\}^*\}$

M_1 = "On input string w:

1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether these positions contain the same symbol. If they do not, or if no # is found, *reject*. Cross off symbols as they are checked to keep track of which symbols correspond.

Infinite memory (initial contents are the **input string**)

Can move to, and **read/write from** <u>arbitrary</u> **memory locations!**