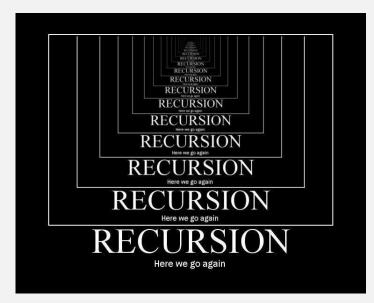
UMass Boston Computer Science **CS450 High Level Languages**

Generative Recursion

Tuesday, March 11, 2025

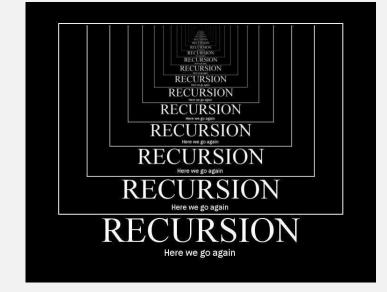




Logistics

- HW 5 in
 - Due: Tues 3/11 11am EST
- HW 6 out
 - Due: **Tues 3/25 11am EST** (2 weeks)
- Reminder: Spring Break next week!
 - No lecture

(improper base case!)





Htdp ch23

Two-Argument Templates

- Sometimes ... a fn must process two arguments simultaneously
- This template should combine templates of both args
 - (This is only possible if the data defs are simple enough)

Must combine all these pieces together somehow ...

```
;; hit?: Ball Ball -> Boolean?
;; evaluates to true if the two Balls have overlap
```

```
(define (hit? b1 b2)
... (ball-x b1) (ball-y b1) ...
(ball-x b2) (ball-y b2) ... )
```

Must combine all these pieces together somehow ...

```
;; hit?: Ball Ball -> Boolean?
;; evaluates to true if the two Balls have overlap
```

Must combine all these pieces together somehow ...

```
;; hit?: Ball Ball -> Boolean?
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```

```
;; hit?: Ball Ball -> Boolean?
;; evaluates to true if the two Balls have overlap
```

Recursion review

Most recursion is structural (i.e., comes from data definitions)!

```
(define (lst-fn lst)
  (cond
  [(empty? lst) ...]
  [else ... (first lst) ... (lst-fn (rest lst)) ...]))
TEMPLATE
```

A Different Kind of Recursion!

• Not all recursion is structural (i.e., comes from data definitions)!

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A Different Kind of Recursion!

- Non-structural recursion (i.e., doesn't come from data definitions) is called generative recursion
- no template? ... requires Termination Argument
 - Explains why the function terminates because recursive call is "smaller"!

- 1. Name, Signature
- 2. Description
 - Must include Termination Argument
- 3. Examples
 - Even more important now!
- 4. Code (No structural template, but can use a "general" template)

5. Tests

- 1. Name, Signature
- 2. Description
 - Must include Termination Argument
- 3. Examples
 - Even more important now!
- 4. Code (No structural template, but can use a "general" template)
 - a) Break problems into smaller problems to (recursively) solve
 - b) Determine how to combine smaller solutions
 - c) "trivially solvable" problem is base case!
- 5. Tests

- 4. Code (No structural template, but can use a "general" template)
 - a) Break problems into smaller problems to (recursively) solve
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 - a) Break problems into smaller problems to (recursively) solve
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```
;; genrec-algo: ??? -> ???
(define (genrec-algo problem)
  (cond
             problem) (solve-easy problem)
   [else (combine-solutions
           (genrec-algo (create-smaller-1 problem))
           (genrec-algo (create-smaller-n problem)))]))
```

- 4. Code (No structural template, but can use a "general" template)
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 - a) Break problems into smaller problems to (recursively) solve
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 - c) "trivially solvable" problem is base case!

```
;; genrec-algo: ??? -> ???
;; termination argument: recursive calls are "smaller" bc ...
(define (genrec-algo problem)
  (cond
   [(trivial? problem) (solve-easy problem)] ;; base case
   [else (combine-solutions
           (genrec-algo (create-smaller-1 problem))
           (genrec-algo (create-smaller-n problem)))]))
```

GenRec Template Generalizes Structural!

```
    Trivial solution = data def base case

(define (lst-fn lst)

    Recursive smaller problem = data def smaller piece

  (cond

    Left to figure out "Combining" pieces

   [(empty? lst) ...]
   [else ... (first lst) ...
                             (lst-fn (rest lst)) ...]))
;; genrec-algo: ??? -> ??/
(define (genrec-algo problem)
  (cond
   [(trivial? problem) (solve-easy problem)];; base case
   [else (combine-solut/ions
             (genrec-algo (create-smaller-1 problem))
             (genrec-algo (create-smaller-n problem))))))
```

Previously

Generative Recursion Example!

(check-equal?

(Functional) Quicksort

;; smaller-than: ListofInt Int -> ListofInt

```
(smaller-than (list 1 3 4 5 9) 4)
  Returns a list containing elements of given list
                                                                (list | 1 3))
;; that are <u>less than</u> the given int
;; larger-than: ListofInt Int -> ListofInt
                                                    (check-equal?
                                                      (greater-than (list 1 3 4 5 9) 4)
  Returns a list containing elements of given list
                                                                 (list 5 9))
;; that are greater than the given int
;; qsort: ListofInt -> ListofInt
;; sorts the given list of ints in ascending order
(define (qsort lst)
   (define pivot (random lst))
   (append (qsort (smaller-than lst pivot))
            (list pivot)
            (qsort (greater-than lst pivot))))
```

Quicksort overview ("divide and conquer")

{10, 80, 30, 90, 40, 50, 70}

- 1. <u>Choose</u> "pivot" element
- 2. Partition into smaller lsts:
 - < pivot</pre>
 - >= pivot
- 3. Recurse on smaller lists
 - Until base case
- 4. <u>Combine</u> small solutions

Choose "pivot" element

Partition into smaller lsts:

```
< pivot
;; qsort: List<Int> -> List<Int>
                                              >= pivot
  termination argument:
                                             Recurse until base case
(define (qsort 1st)
                                             Combine small solutions
  (cond
   [(trivial? problem) (solve-easy lst)] ;; base case
   [else
    (define pivot (first lst))
    (combine-solutions
      (qsort (smaller-problem-1 lst))
      (qsort (smaller-problem-n lst)))])
```

```
Choose "pivot" element
                                                        Partition into smaller lsts:
                                                        • < pivot</pre>
      qsort: List<Int> -> List<Int>
                                                        • >= pivot
      terminati Function "arithmetic"!
                                                     3. Recurse until base case
                  (curry f arg1)
Result is a function!
                                                        Combine small solutions
         (lambda (arg2) (f arg1 arg2))
                                                            (curry > pivot)
       [e]se
 Curry = "partial apply"
                        (first lst))
                                                       (lambda (\underline{x}) (> pivot \underline{x}))
        (combine-solutions
           (qsort (filter (curry > pivot) (rest 1st)) "less than"
           (qsort (filter (curry <= pivot) (rest lst) "greater than"
```

Choose "pivot" element

```
Partition into smaller lsts:
                                                 • /< pivot</pre>
  qsort: List<Int> -> List<Int>
                                                  >= pivot
  termination argument:
                                                  Recurse until base case
(define (qsort 1st)
                                                  Combine small solutions
  (cond
   [(trivial? problem) (solve-easy
   [else
    (define pivot (first lst))
    (combine-solutions
      (qsort (filter (curry $ pivot)/(rest 1st)) "less than"
      (qsort (filter (curry <= pivot) (rest lst) "greater than"
```

1. Choose "pivot" element

Partition into smaller lsts:

```
• < pivot</pre>
;; qsort: List<Int> -> List<Int>
                                                >= pivot
;; termination argument:
                                             3. Recurse until base case
(define (qsort 1st)
                                             4. Combine small solutions
  (cond
   [(empty? lst) empty] ;; base case
   [else
    (define pivot (first lst))
    (combine-solutions
      (qsort (filter (curry > pivot) (rest lst)))
      (qsort (filter (curry <= pivot) (rest lst))))))</pre>
```

Choose "pivot" element

Partition into smaller lsts:

```
• < pivot</pre>
;; qsort: List<Int> -> List<Int>
                                                >= pivot
;; termination argument:
                                             3. Recurse until base case
(define (qsort 1st)
                                             4. <u>Combine</u> small solutions
  (cond
   [(empty? lst) empty] ;; base
   [else
    (define pivot (first lst))
    (append
      (qsort (filter (curry > pivot) (rest lst)))
      (list pivot)
      (qsort (filter (curry <= pivot) (rest lst))))))
```

```
;; qsort: List<Int> -> List<Int>
;; termination argument:
  recursive calls "smaller" bc at least one item dropped (pivot)
(define (qsort 1st)
  (cond
   [(empty? lst) empty] ;; base case
   [else
    (define pivot (first lst))
    (append
      (qsort (filter (curry > pivot) (rest lst)))
      (list pivot)
      (qsort (filter (curry <= pivot) (rest lst))))))
```

Interlude: Recursion vs Iteration

• Recursive functions have a self-reference

```
def factorialUsingRecursion(n):
    if (n == 0):
       return 1;

# recursion call
    return n * factorialUsingRecursion(n - 1);
```

Iterative code typically use a loop

```
def factorialUsingIteration(n):
    res = 1;

    # using iteration
    for i in range(2, n + 1):
        res *= i;

    return res;
```

Recursion vs Iteration: Which is "Better"?

Recursive vs. Iterative Solutions

Recursive algorithms can be very space inefficient. Each recursive call adds a new layer to the stack, which means that if your algorithm recurses to a depth of n, it uses at least O(n) memory.

For this reason, it's often better to implement a recursive algorithm iteratively. *All* recursive algorithms can be implemented iteratively, although sometimes the code to do so is much more complex. Before diving into recursive code, ask yourself how hard it would be to implement it iteratively, and discuss the tradeoffs with your interviewer.

Cracking the Coding Interview, Ch8



[Best Practices] Recursion. Why is it generally avoided and when is it acceptable?



Are recursive methods always better than iterative methods in Java?

Recursion vs Iteration: Conventional Wisdom

Strengths: Iteration

- Iteration can be used to repeatedly execute a set of statements without the overhead of function calls and without using stack memory.
- Iteration is faster and more efficient than recursion.
- It's easier to optimize iterative codes, and they generally have polynomial time complexity.
- They are used to iterate over the elements present in data structures like an array, set, map, etc.
- If the iteration count is known, we can use *for* loops; else, we can use *while* loops, which terminate when the controlling condition becomes false.

Weaknesses:

- In loops, we can go only in one direction, i.e., we can't go or transfer data from the current state to the previous state that has already been executed.
 It's difficult to traverse trees/graphs using loops.

 It's difficult to traverse trees/graphs using loops.
- Only limited information can be passed from one iteration to another, while in recursion,
 we can pass as many parameters as we need.
 Recursion better when accumulators are needed

Iteration is **good** with **non-recursive data**

https://www.interviewkickstart.com/learn/difference-between-recursion-and-iteration

Recursion vs Iteration: Conventional Wisdom

Strengths: Recursion

- It's easier to code the solution using recursion when the solution of the current problem is dependent on the solution of smaller similar problems.
 - fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
 - factorial(n) = n * factorial(n-1)



Recursive codes are smaller and easier to understand.

• We can pass information to the next state in the form of parameters and return information to the previous state in the form of the return value.

• It's a lot easier to perform operations on trees and graphs using recursion.

Use recursion with recursive data!

Recursion is slow • The simple

The simplicity of recursion comes at the cost of time and space efficiency.

Recursion is slow

• It is much slower than iteration due to the overhead of function calls and control shift from one function to another.

Recursion is slow

• It requires extra memory on the stack for each recursive call. This memory gets deallocated when function execution is over.

Recursion is slow

Investigate:

Is recursion is slower??

Recursion better

are needed

when **accumulators**

s difficult to optimize a recursive code, and they generally have higher time complexity an iterative codes due to overlapping subproblems.

https://www.interviewkickstart.com/learn/difference-between-recursion-and-iteration

Recursion vs Iteration: In Racket

```
Racket Recursion
;; sum-to : Nat -> Nat
;; Sums the numbers in the interval [0, x]
                                                    Conclusion?
(define (sum-to x)
                                                    Recursion is slower?
  (if (zero? x)
      X
                                             WAIT!
      (+ x (sum-to (sub1 x)))))
                                             Racket does not have "for" loops
     (define BIG-NUMBER 999999)
     (time (sum-to BIG-NUMBER))
     ; cpu time: 202 real time: 201 gc time: 156
                                                                  Racket "Iteration"
                                 (time (for/sum ([x (add1 BIG-NUMBER)]) x))
                                 ; cpu time: 15 real time: 6 gc time: 0
```

Recursion vs Iteration: In Racket

Conclusion?

Recursion is <u>not</u> slower than iteration?

equivalent

Racket Recursion

"for" in Racket is just a macro (i.e., "syntactic sugar") for a recursive function

```
Racket "Iteration"

(time (for/sum ([x (add1 BIG-NUMBER)]) x))

; cpu time: 15 real time: 6 gc time: 0
```

Tail Calls

From Wikipedia, the free encyclopedia

In computer science, a **tail call** is a subroutine call performed as the final action of a procedure. If the target of a tail is the same subroutine, the subroutine is said to be **tail recursive**, which is a special case of direct recursion. **Tail recursion** (or **tail-end recursion**) is particularly useful, and is often easy to optimize in implementations.

Tail calls can be implemented without adding a new stack frame to the call stack.

Recursion vs Iteration: In Racket

Conclusion?

Recursion is <u>not</u> slower than iteration?

```
Racket Recursion
```

Tail-call (does not add to stack)

(Tail) recursion is iteration!

Recursion vs Iteration: Under the Hood

- It makes sense that recursion and iteration are equivalent ...
 - Recursive call compiles to:
 - **JUMP** instruction
 - Loop compiles to:
 - JUMP instruction!
- ... except in languages that make them not equivalent!
 - i.e., languages that push extra stack frames that are not needed

Tail-Calls in Other Languages

• Most functional languages (RACKET, HASKELL, ERLANG, F#) implement proper tail calls (no extra stack frame)

- Some languages require an explicit annotation
 - CLOJURE: recur
 - SCALA: @tailrec
- Some languages (JAVASCRIPT) have it (ECMASCRIPT 6), but don't have it
- Most imperative languages don't properly implement tail calls (they add an unnecessary stack frame)
 - Python, Java, C#, Go

Guido Got It Backwards

Wednesday, April 22, 2009

Tail Recursion Elimination

I recently posted an entry in my Python History blog on the origins of Python's functional features. A side remark about not supporting tail recursion elimination (TRE) immediately sparked several comments about what a pity it is that Python doesn't do this, including links to recent blog entries by others trying to "prove" that TRE can be added to Python easily. So let me defend my position (which is that I don't want TRE in the language). If you want a short answer, it's simply unpythonic. Here's the long answer:

Wrong!

First, as one commenter remarked, TRE is incompatible with nice stack traces: when a tail recursion is eliminated, there's no stack frame left to use

to print a traceback when something goes wrong later. This will confuse Equivalent to saying: ently wrote something recursive (the recursion isn't obviou "every **for** loop iteration ce printed), and makes debugging hard. Providing an optio should push a stack frame!" seems wrong to me: Python's default is and should always b be maximally helpful for debugging. This also brings me to the next issu place! (because it's just iteration!)

About Me



Guido van Rossum

Python's BDFL

View my complete profile

Blog Archive

2022 (2)

2019 (1)

Proper tail calls is about **eliminating stack frames** that shouldn't be there in the first

Non Tail Call

```
// Program to find factorial of a number n modulo prime
int factorial(int n, int prime)

if (n <= 1) {
    // base case
    return 1;
    }

return (n * factorial(n - 1, prime) % prime) % prime;

Non-tail-call

Slower, more memory</pre>
```

```
factorial:
 1
        cmp edi, 1
        jle <u>.L16</u>
        push r15
        mov eax, 2
        push r14
        push r13
        push r12
        lea r12d, [rdi-1]
        push rbx
        mov ebx, edi
11
        sub rsp, 16
12
        cmp edi, 2
13
        je <u>.L3</u>
14
       lea r13d, [rdi-2]
15
        cmp edi, 3
16
        je <u>.L4</u>
17
       lea r15d, [rdi-3]
18
        cmp edi, 4
19
20
        je <u>.L5</u>
       lea r14d, [rdi-4]
21
        cmp edi, 5
22
23
        je <u>.L6</u>
        lea edi, [rdi-5]
24
        mov DWORD PTR [rsp+12], esi
       call <u>factorial</u>
                                          Stack push
26
        mov esi, DWORD PTR [rsp+12]
27
        imul eax, r14d
28
        cdq
29
        idiv esi
30
        mov eax, edx
31
32
       imul eax, r15d
33
      .L6:
        cdq
34
        idiv esi
35
        mov eax, edx
36
       imul eax, r13d
37
      .L5:
38
        cdq
39
        idiv esi
40
        mov eax, edx
41
       imul eax, r12d
      .L4:
43
        cdq
        idiv esi
```

Compiler output: godbolt.org

x86-64 gcc 14.2





https://www.geeksforgeeks.org/tail-call-optimisation-in-c/

x86-64 gcc 14.2



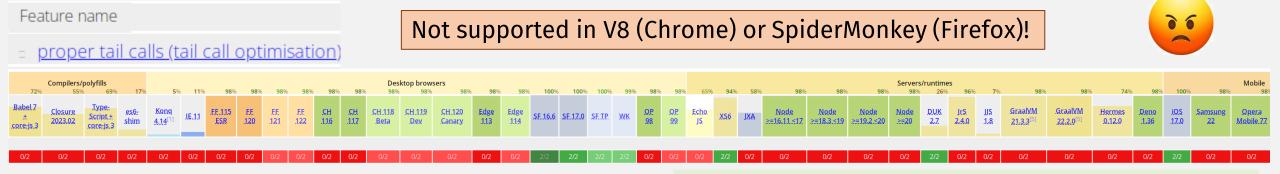
Tail Calls as Loops

```
C program to illustrate Tail Call Optimisation
                                                                                      // this function calculates factorial modulo prime
  int factorial(int store, int num, int prime) {
                                                                                      int factorial(int num, int prime) {
    if (num < 1) {
                                                                                          int store = 1;
      // Base case
                                                                                          for (int i = num; i > 0; i--) {
      return store;
                                                                                              store = (store%prime * i%prime)%prime;
                                                                                                           for loop
                                                                                        return store;
    return factorial((store%prime * num%prime)%prime, num - 1, prime);
      Tail call – compiled directly to for loop!
        Faster, less memory
                                                            factorial(int, int, int):
                                                                                        factorial(int, int):
                                                             mov ecx, edx
                                                                                          mov edx, 1
                                                             test esi, esi
                                                                                          test edi, edi
                                                             jle <u>.L5</u>
                                                                                          jle <u>.L1</u>
                                                            .L2:
                                                                                         .L3:
                                                             mov eax, edi
                                                                                          mov eax, edx
                                                             cdq
                                                                                          cdq
                                       No stack push!
                                                             idiv ecx
                                                                                          idiv esi
                                                                                                              Recursion is same
                                                             mov eax, edx
                                                                                          mov eax, edx
                                                                                                              as for loop!
                                                             imul eax, esi
                                                                                          imul eax, edi
                                                                                    10
                                                       11
                                                              cdq
                                                                                          cdq
                                                                                    11
                                                             idiv ecx
                                                                                          idiv esi
                                                       12
                                                             mov edi, edx
                                                                                          sub edi, 1
                                                       -13
Some languages (with -O3 optimization)
                                                             sub esi, 1
                                                       14
                                                                                          jne <u>.L3</u>
                                                                                    14
                                                             jne <u>.L2</u>
                                                                                         .L1:
                                                                                    15
directly compile recursion to a loop!
                                                       16
                                                            .L5:
                                                                                    16
                                                                                          mov eax, edx
                                                             mov eax, edi
                                                                                           ret
(because they are equivalent!)
                                                             ret
```

Proper Tail Calls in JavaScript

Proper Tail Calls (PTC) is a new feature in the ECMAScript 6 language. This feature was added to facilitate recursive programming patterns, both for direct and indirect recursion. Various other design patterns can benefit from PTC as well, such as code that wraps some functionality where the wrapping code directly returns the result of what it wraps. Through the use of PTC, the amount of memory needed to run code is reduced. In deeply recursive code, PTC enables code to run that would otherwise throw a stack overflow exception.

https://webkit.org/blog/6240/ecmascript-6-proper-tail-calls-in-webkit/



Recursion vs Iteration: Conclusion

Recursion Strengths:

- It's easier to code the solution using recursion when the solution of the current problem is dependent on the solution of smaller similar problems.
 - fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
 - factorial(n) = n * factorial(n-1)

Recursion is (usually) easier to read

- Recursive codes are smaller and easier to understand.
- We can pass information to the next state in the form of parameters and return information to the previous state in the form of the return value.
- It's a lot easier to perform operations on trees and graphs using recursion

Use recursion with recursive data!

Weaknesses:

- The simplicity of recursion comes at the cost of time and space efficiency.
- It is much slower than iteration due to the overhead of function calls and control shift

from one function to another.

- t requires extra memory on the stack for each recursive call. This memory gets deallocated when function execution is over.
- t is difficult to optimize a recursive code, and they generally have higher time complexity han iterative codes due to overlapping subproblems.

Recursion better when accumulators are needed

Recursion is slower ...

... in languages that choose to make it slower!

https://www.interviewkickstart.com/learn/difference-between-recursion-and-iteration