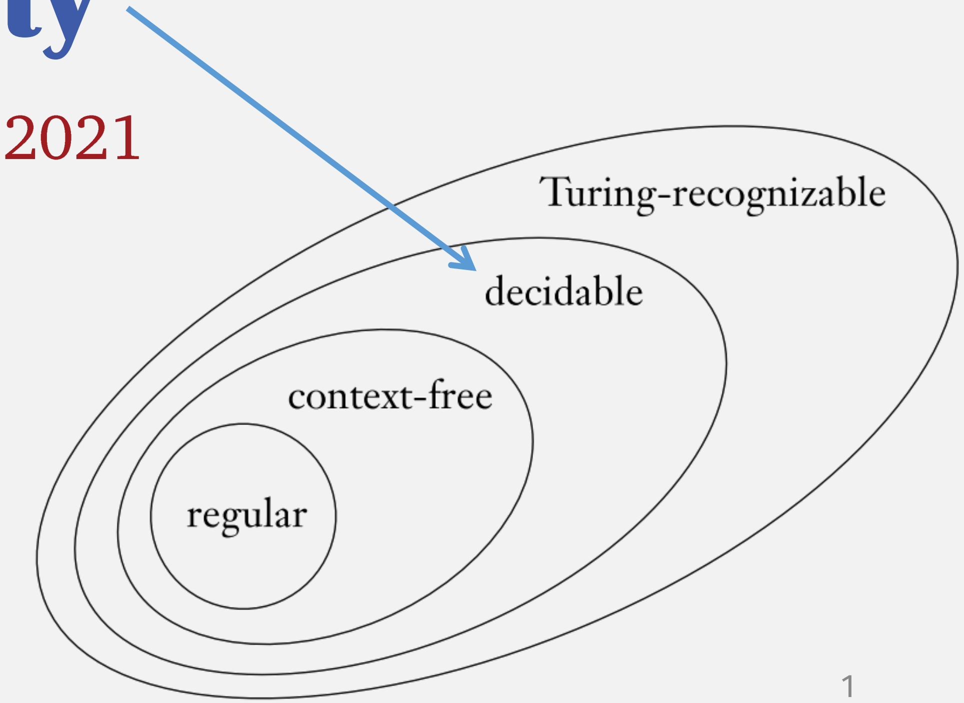
UMB CS622

Decidability

Monday, October 18, 2021

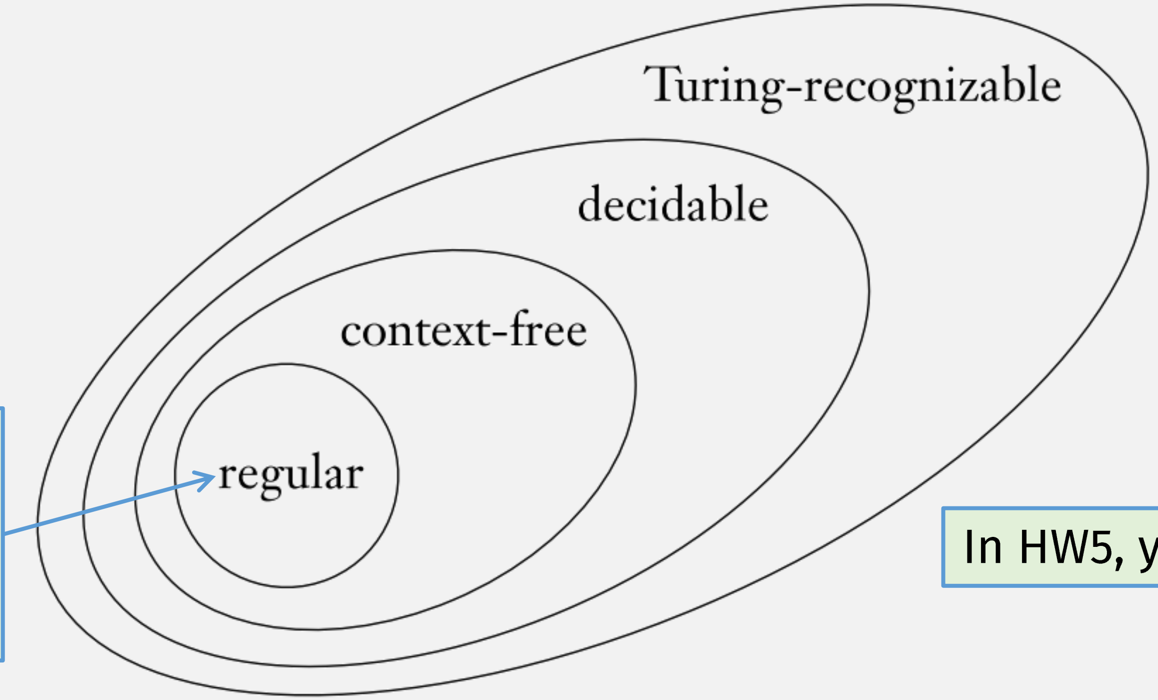


Announcements

- HW4 in

- HW5 out
    - Due Sun 10/24 11:59pm

Correctness of this Diagram?



HW4, Problem 5 **proved that “regular” circle is correctly inside “context-free” circle**

In HW5, you’ll prove the rest

Turing Machines and Algorithms

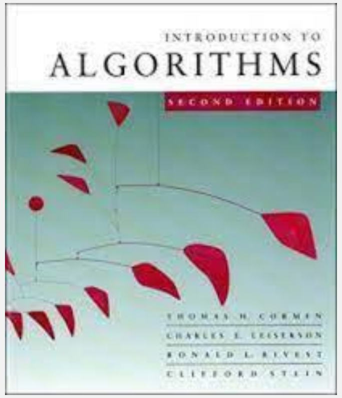
- Turing Machines can express any "computation"
  - I.e., **a Turing Machine models** (Python, Java) **programs**!

- 2 classes of Turing Machines
  - Recognizers may loop forever
  - Deciders always halt

Today

- Deciders = Algorithms
  - I.e., an algorithm is any program that always halts

Remember:
TMs = programs



Algorithms (Decidable Problems) for Regular Languages

$\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***

Flashback: Running a DFA "Program"

Define the extended transition function: $\hat{\delta} : Q \times \Sigma^* \to Q$

Base case: $\hat{\delta}(q, \epsilon) = q$

First char

Last chars

Remember:
TMs = programs

Recursive case: $\hat{\delta}(q, a_1 w_{rest}) = \hat{\delta}(\delta(q, a_1), w_{rest})$

Single transition step

Could you implement this as a program?

A function `DFAaccepts(B,w)` that returns TRUE if DFA `B` accepts string `w`

- Define "current" state q_{current} = start state q_0
- For each input char a_i …
    - Define $q_{\text{next}} = \delta(q_{\text{current}}, a_i)$
    - Set $q_{\text{current}} = q_{\text{next}}$
- Return TRUE if q_{current} is an accept state

The language of `DFAaccepts`

Function `DFAaccepts(B,w)`
returns TRUE if DFA `B` accepts string `w`

$$A_{\mathsf{DFA}} = \{\langle B, w\rangle \mid B \text{ is a } \mathsf{DFA} \text{ that accepts input string } w\}$$

But a language is a set of strings?

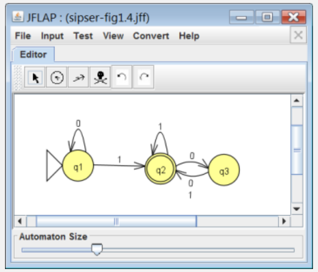
Interlude: Encoding Things into Strings

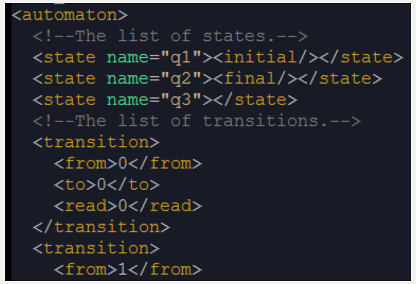
- A Turing machine’s input is always a string

- So anything we want to give to TM must be **encoded** as string

Notation: <SOMETHING> = string encoding for SOMETHING
  - A tuple combines multiple encodings, e.g., $<B, w>$ (from prev slide)

Example: Possible string encoding for a DFA?





```
<automaton>
  <!--The list of states.-->
  <state name="q1"><initial/></state>
  <state name="q2"><final/></state>
  <state name="q3"></state>
  <!--The list of transitions.-->
  <transition>
    <from>0</from>
    <to>0</to>
    <read>0</read>
  </transition>
  <transition>
    <from>1</from>
```

Interlude: Informal TMs and Encodings

An informal TM description:

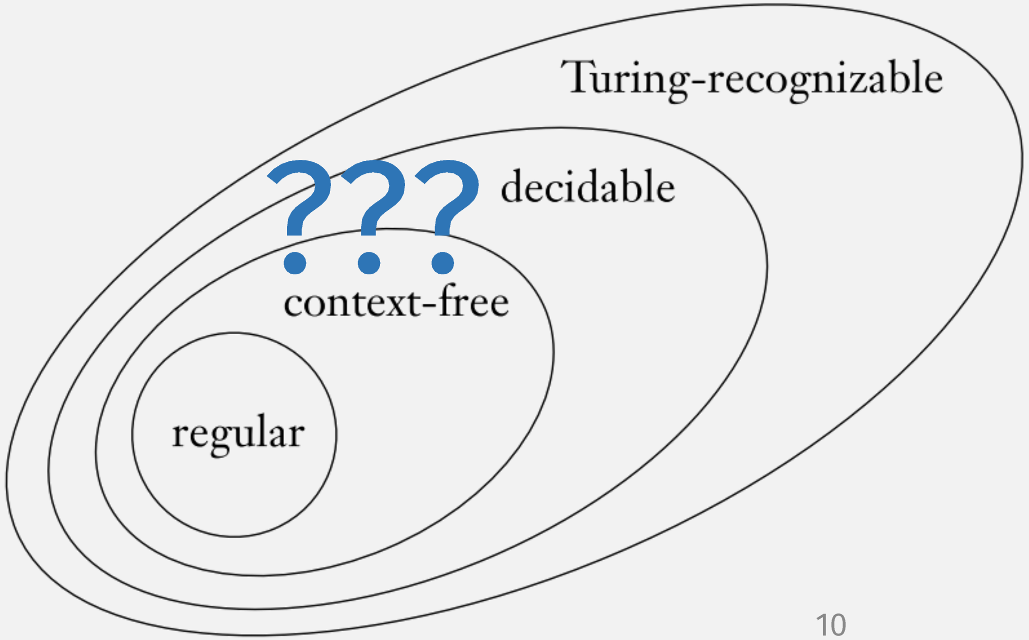
1. Doesn’t need to describe exactly how input string is encoded
2. Assumes input is a “valid” encoding
   - Invalid encodings are automatically rejected

The language of **`DFAaccepts`**

$$A_{\mathsf{DFA}} = \{\langle B, w\rangle \mid B \text{ is a } \mathsf{DFA} \text{ that accepts input string } w\}$$

- **`DFAaccepts`** is a Turing machine
- But is it a decider or recognizer?
  - I.e., is it an algorithm?
- To show it's an algo, need to prove:

$$A_{\mathsf{DFA}} \text{ is a decidable language}$$



How to prove that a language is decidable?

- Create a Turing machine that decides that language!

Remember:

- A decider is Turing Machine that always halts
  - I.e., for any input, either accepts or rejects it.

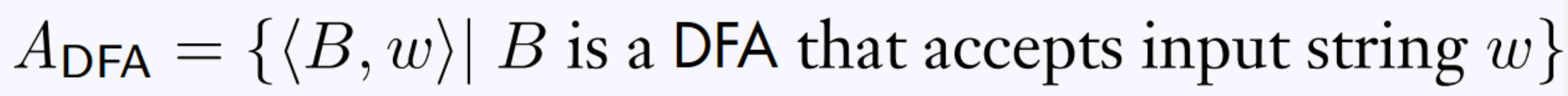
How to Design Deciders

- If TMs = Programs …

  … then **Creating** a TM = Programm**ing**

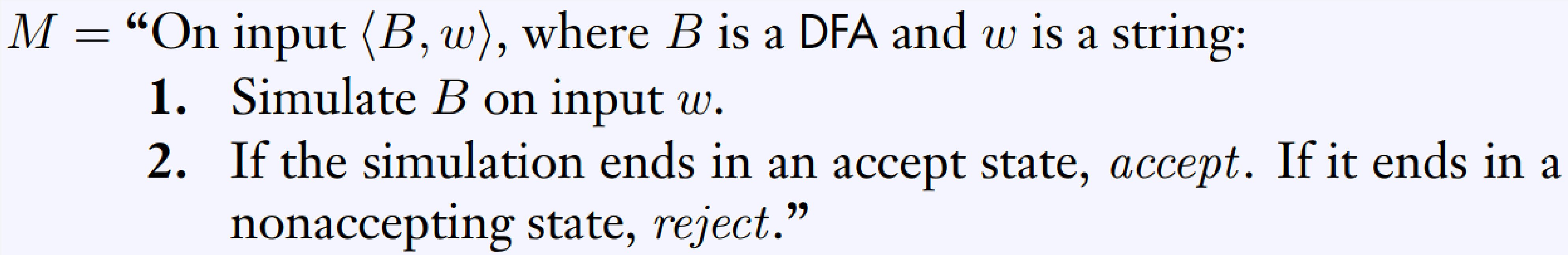
- E.g., if HW asks “Show that lang L is decidable” …
  - .. you must create a TM that decides L; to do this …
  - … think of how to write a (halting) program that does what you want

Thm: A_{DFA} is a decidable language



$A_{\text{DFA}} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts input string } w\}$

Decider for A_{DFA} :



```
$M$ = "On input $\langle B, w\rangle$, where $B$ is a DFA and $w$ is a string:
    1. Simulate $B$ on input $w$.
    2. If the simulation ends in an accept state, *accept*. If it ends in a
       nonaccepting state, *reject*."
```

Where "Simulate" =

- Define "current" state q_{current} = start state q_0
- For each input char x …
  - Define $q_{\text{next}} = \delta(q_{\text{current}}, x)$
  - Set $q_{\text{current}} = q_{\text{next}}$

Remember:
TMs = programs
Creating TM = programming

Termination Argument: **This is a decider (i.e., it always halts) because the input is always finite, so the loop always terminates**

Deciders must also have a **termination argument:**
Explains how every step in the TM halts (we typically only care about loops)

Thm: A_{NFA} is a decidable language

$A_{\mathsf{NFA}} = \{\langle B, w\rangle \mid B$ is an NFA that accepts input string $w\}$

Decider for A_{NFA} :

Flashback: NFA→DFA

Have: $N = (Q, \Sigma, \delta, q_0, F)$

Want to: construct a DFA $M = (Q', \Sigma, \delta', {q_0}', F')$

1. $Q' = \mathcal{P}(Q)$.

2. For $R \in Q'$ and $a \in \Sigma$,

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$$

3. ${q_0}' = \{q_0\}$

4. $F' = \{R \in Q' | \, R \text{ contains an accept state of } N\}$

Could you implement this conversion algorithm as a program?

This is a Turing Machine

Thm: A_{NFA} is a decidable language

$$A_{\text{NFA}} = \{\langle B, w\rangle \mid B \text{ is an NFA that accepts input string } w\}$$

Decider for A_{NFA} :

```
$N$ = "On input $\langle B, w\rangle$, where $B$ is an NFA and $w$ is a string:
    1. Convert NFA $B$ to an equivalent DFA $C$, using the procedure
       NFA→DFA
    2. Run TM $M$ on input $\langle C, w\rangle$.  ($M$ is $A_{\text{DFA}}$ decider from prev slide)
    3. If $M$ accepts, accept; otherwise, reject."
```

Remember:
TMs = programs
Creating TM = programming
Previous theorems = library

Termination argument: **This is a decider (i.e., it always halts) because:**

- **Step 1 always halts bc there's a finite number of states in an NFA**
- **Step 2 always halts because M is a decider**

How to Design Deciders, Part 2

- If TMs = Programs …

  … then **Creating** a TM = Programm**ing**

- E.g., if HW asks “Show that lang L is decidable” …
  - .. you must create a TM that decides L; to do this …
  - … think of how to write a (halting) program that does what you want

<u>Hint</u>:

- Previous theorems are a “library” of reusable TMs
- When creating a TM, try to use these theorems to help you
  - Just like you use <u>libraries</u> when programming!
- E.g., “Library” for DFAs:
  - `NFA→DFA`, `RegExpr→NFA`,
  - union, intersect, star, decode, reverse
  - Deciders for: A_{DFA}, A_{NFA}, A_{REX}, …

Thm: A_{REX} is a decidable language

$A_{\text{REX}} = \{\langle R, w\rangle \mid R \text{ is a regular expression that generates string } w\}$

Decider:

$P =$ "On input $\langle R, w\rangle$, where R is a regular expression and w is a string:

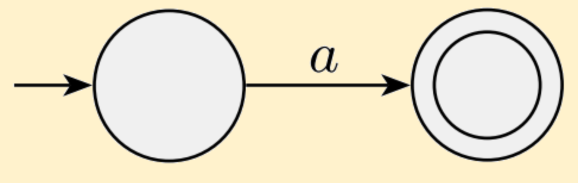
1. Convert regular expression R to an equivalent NFA A by using the procedure `RegExpr→NFA`

RegExpr→NFA

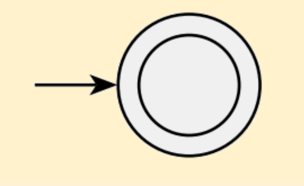
Does this conversion always halt?

R is a ***regular expression*** if R is

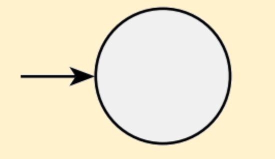
1. a for some a in the alphabet Σ,



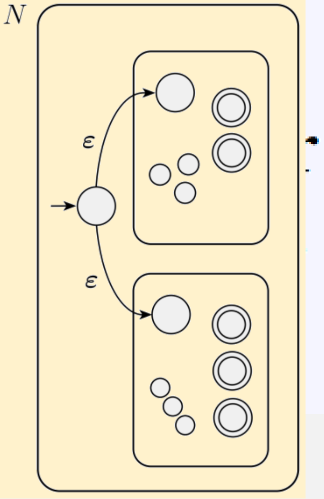
2. ε,

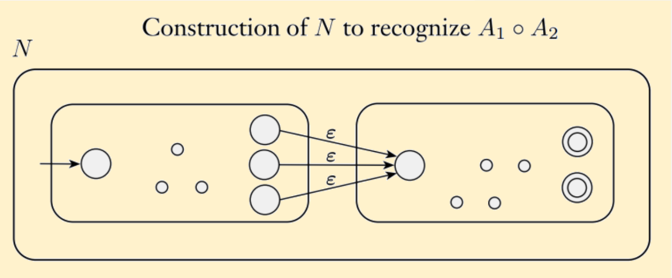


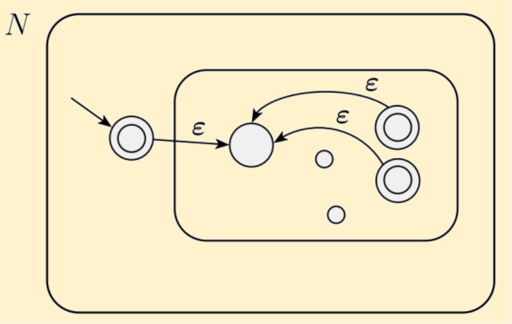
3. $\emptyset$,



4. $(R_1 \cup R_2)$, where R_1 and R_2 a r e
5. $(R_1 \circ R_2)$, where R_1 and R_2 ar expressions, or
6. (R_1^*), where R_1 is a regular exp





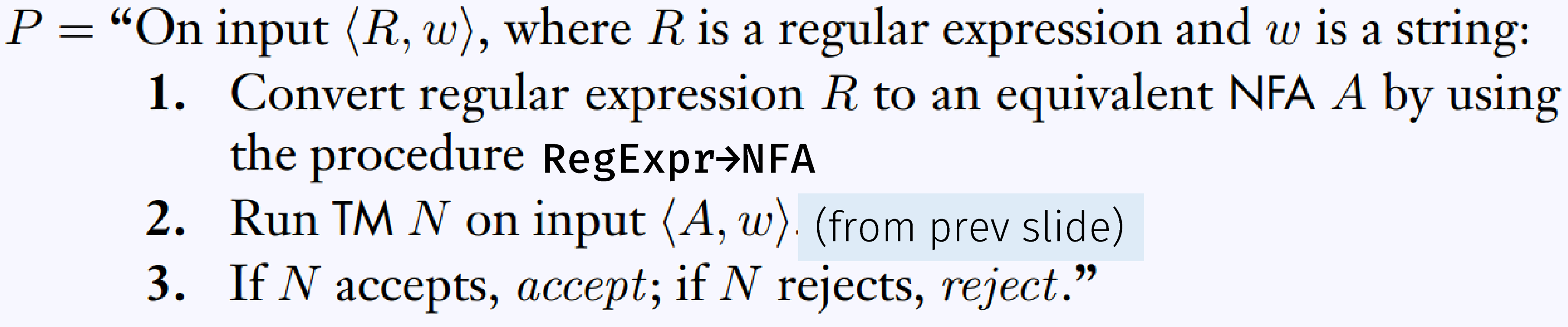


Yes, because recursive call only happens on “smaller” reg exprs

Thm: A_{REX} is a decidable language

$A_{\mathsf{REX}} = \{\langle R, w\rangle \mid R \text{ is a regular expression that generates string } w\}$

Decider:



```
P = “On input $\langle R, w\rangle$, where $R$ is a regular expression and $w$ is a string:
    1. Convert regular expression $R$ to an equivalent NFA $A$ by using
       the procedure RegExpr→NFA
    2. Run TM $N$ on input $\langle A, w\rangle$. (from prev slide)
    3. If $N$ accepts, accept; if $N$ rejects, reject.”
```

Termination Argument: **This is a decider because:**

- Step 1 always halts because converting a reg expr to NFA is done recursively, where the reg expr gets smaller at each step, eventually reaching the base case
- Step 2 always halts because *N* is a decider

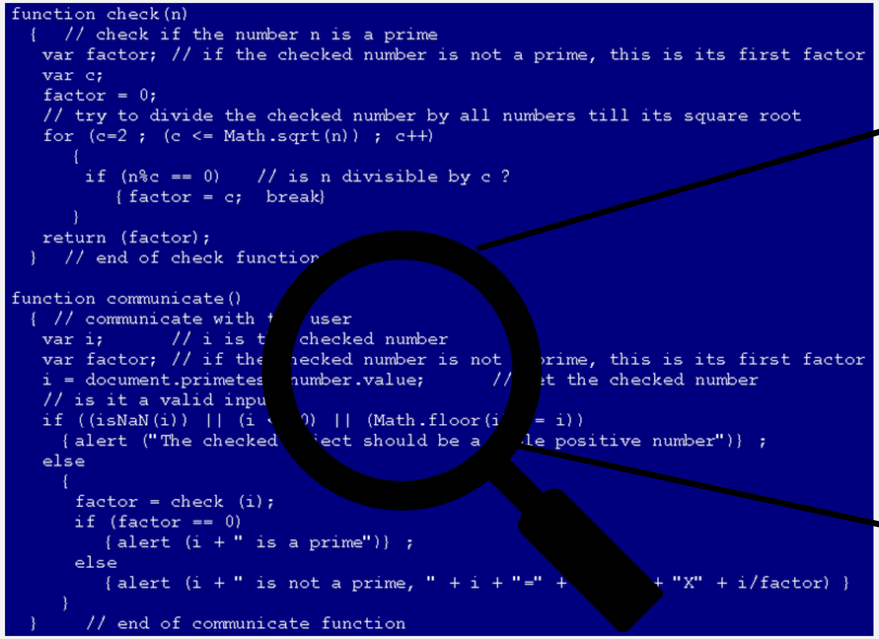
DFA TMs Recap (So Far)

Remember:
TMs = programs
Creating TM = programming
Previous theorems = library

- $A_{\mathsf{DFA}} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts input string } w\}$
  - Deciding TM implements extended DFA δ
- $A_{\mathsf{NFA}} = \{\langle B, w\rangle \mid B \text{ is an NFA that accepts input string } w\}$
  - Deciding TM uses **NFA→DFA** + DFA decider
- $A_{\mathsf{REX}} = \{\langle R, w\rangle \mid R \text{ is a regular expression that generates string } w\}$
  - Deciding TM uses **RegExpr→NFA** + **NFA→DFA** + DFA decider

Flashback: Why study computers formally?

2. **To predict what programs will do**
    - (without running them!)



```
function check(n)
  {   // check if the number n is a prime
   var factor; // if the checked number is not a prime, this is its first factor
   var c;
   factor = 0;
   // try to divide the checked number by all numbers till its square root
   for (c=2 ; (c <= Math.sqrt(n)) ; c++)
      {
       if (n%c == 0)    // is n divisible by c ?
          { factor = c;  break}
      }
   return (factor);
  }  // end of check functio

function communicate()
  {  // communicate with t   user
   var i;       // i is t   checked number
   var factor; // if the   ecked number is not   prime, this is its first factor
   i = document.primetes   number.value;      //   et the checked number
   // is it a valid inpu
   if ((isNaN(i)) || (i    0) || (Math.floor(i   = i))
     {alert ("The checked    ect should be a    le positive number")} ;
   else
     {
      factor = check (i);
      if (factor == 0)
         {alert (i + " is a prime")} ;
      else
         {alert (i + " is not a prime, " + i + "=" +           + "X" + i/factor) }
     }
  }     // end of communicate function
```

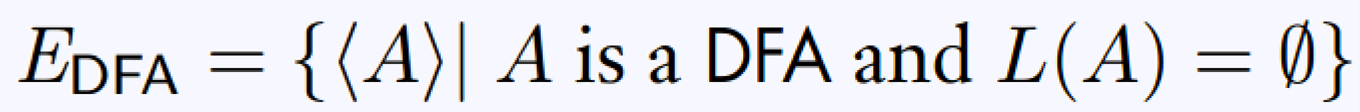
???

Not possible in general! But …

Predicting What <u>Some</u> Programs Will Do …

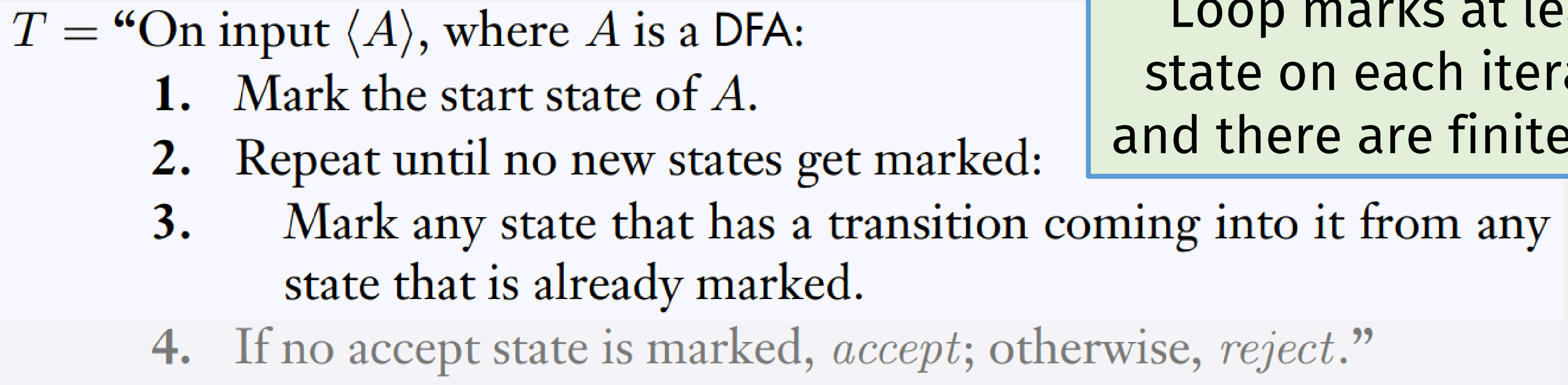
What if we look at weaker computation models
… like DFAs and regular languages!

Thm: E_{DFA} is a decidable language



$E_{\text{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$

Decider:



```
T = "On input $\langle A \rangle$, where $A$ is a DFA:
    1. Mark the start state of $A$.
    2. Repeat until no new states get marked:
    3.     Mark any state that has a transition coming into it from any
           state that is already marked.
    4. If no accept state is marked, accept; otherwise, reject."
```

Loop marks at least 1 state on each iteration, and there are finite states

Termination argument?

I.e., this is a "reachability" algorithm …

… check if accept states are "reachable" from start state

Thm: EQ_{DFA} is a decidable language

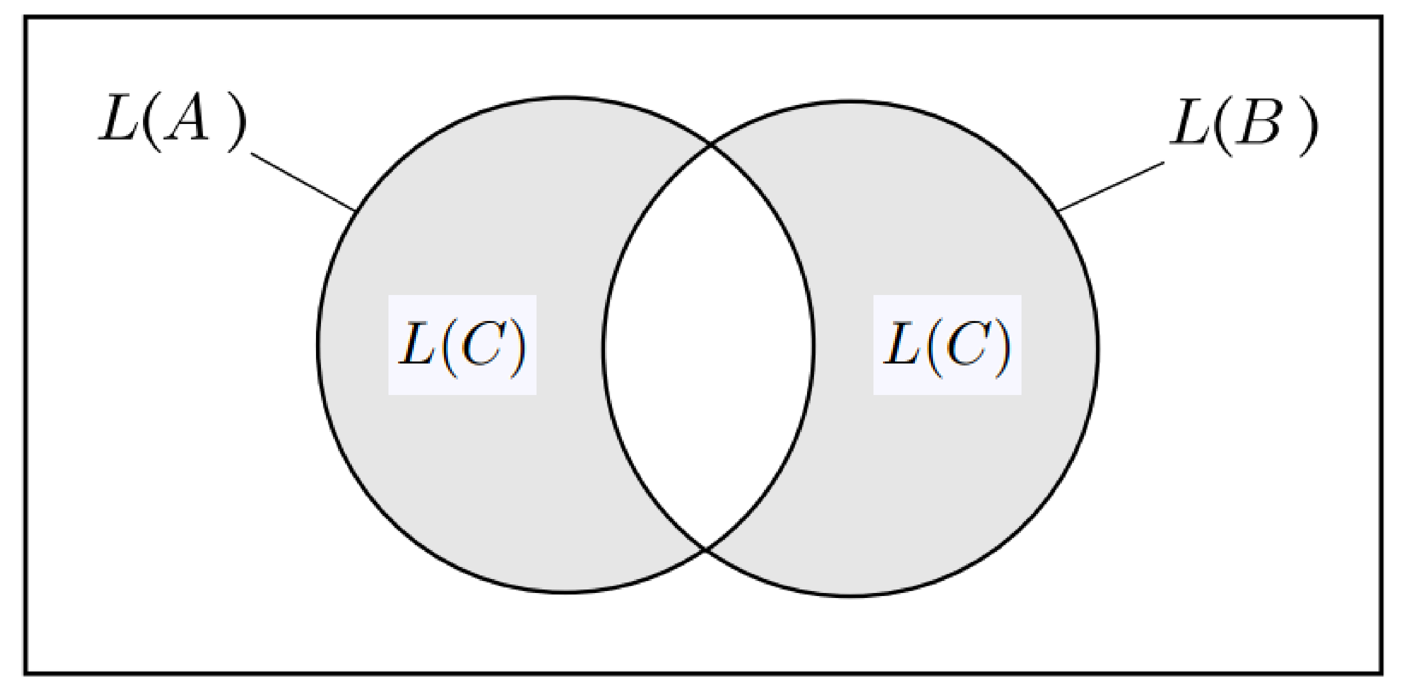
$$EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

I.e., **Can we compute whether two (DFA) programs are "equivalent"?**

(A "holy grail" of computer science)

Trick: Use Symmetric Difference

Symmetric Difference



$$L(C) = \left(L(A) \cap \overline{L(B)}\right) \cup \left(\overline{L(A)} \cap L(B)\right)$$

$$\boxed{L(C) = \emptyset \text{ iff } L(A) = L(B)}$$

Thm: EQ_{DFA} is a decidable language

$$EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

NOTE: This only works because: **negation**, i.e., set complement, and **intersection is closed** for regular languages

Construct decider using 2 parts:

1. Symmetric Difference algo: $L(C) = \left(L(A) \cap \overline{L(B)}\right) \cup \left(\overline{L(A)} \cap L(B)\right)$
   - Construct C = Union, intersection, negation of machines A and B
2. Decider T (from “library”) for: $E_{\text{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$
   - Because $L(C) = \emptyset$ iff $L(A) = L(B)$

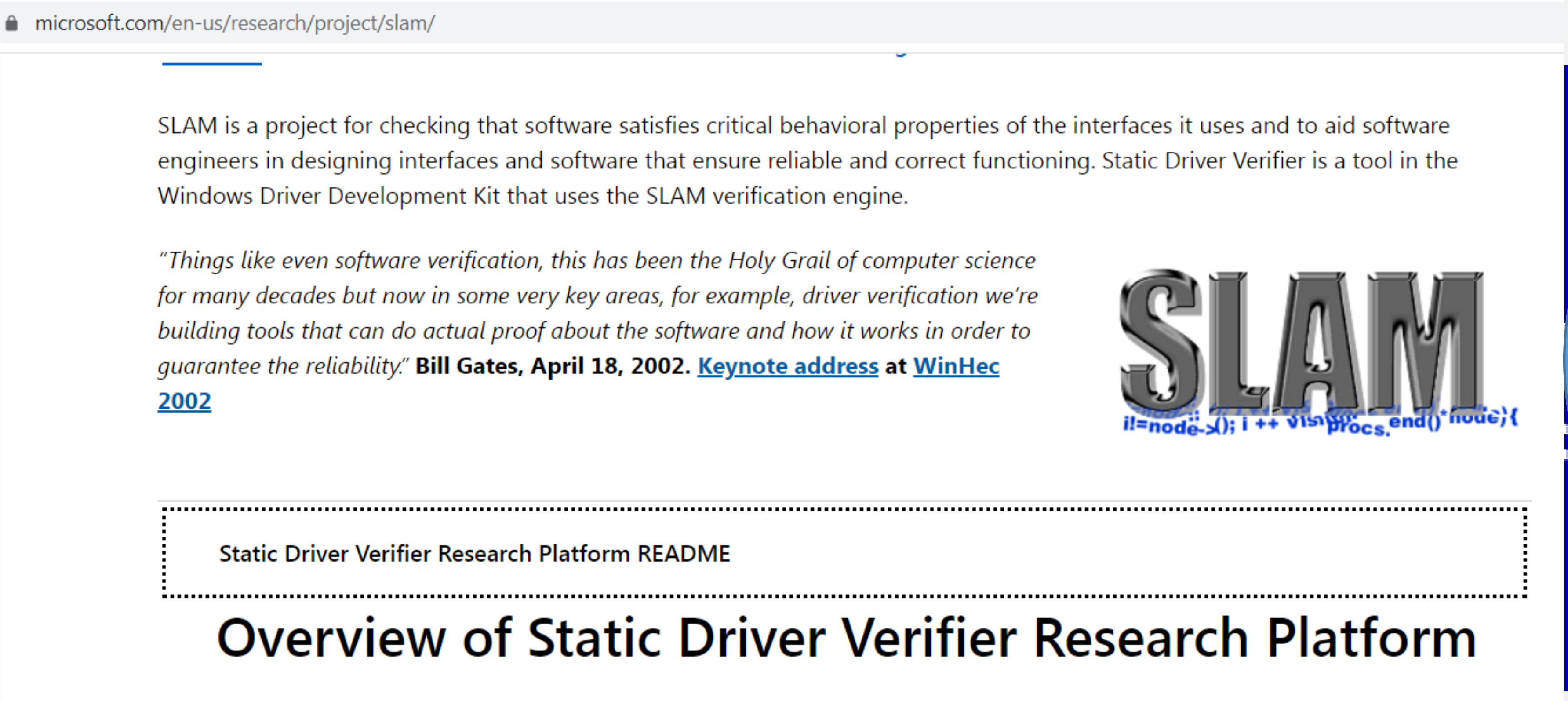
```
F = “On input ⟨A, B⟩, where A and B are DFAs:
    1. Construct DFA C as described.
    2. Run TM T deciding E_DFA on input ⟨C⟩.
    3. If T accepts, accept. If T rejects, reject.”
```

Summary: Decidable DFA Langs (i.e., algorithms)

- $A_{\text{DFA}} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts input string } w\}$
- $A_{\text{NFA}} = \{\langle B, w\rangle \mid B \text{ is an NFA that accepts input string } w\}$
- $A_{\text{REX}} = \{\langle R, w\rangle \mid R \text{ is a regular expression that generates string } w\}$
- $E_{\text{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$
- $EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$

Remember:
TMs = programs
Creating TM = programming
Previous theorems = library

Predicting What Some Programs Will Do …



microsoft.com/en-us/research/project/slam/

SLAM is a project for checking that software satisfies critical behavioral properties of the interfaces it uses and to aid software engineers in designing interfaces and software that ensure reliable and correct functioning. Static Driver Verifier is a tool in the Windows Driver Development Kit that uses the SLAM verification engine.

"Things like even software verification, this has been the Holy Grail of computer science for many decades but now in some very key areas, for example, driver verification we're building tools that can do actual proof about the software and how it works in order to guarantee the reliability." **Bill Gates, April 18, 2002. Keynote address at WinHec 2002**

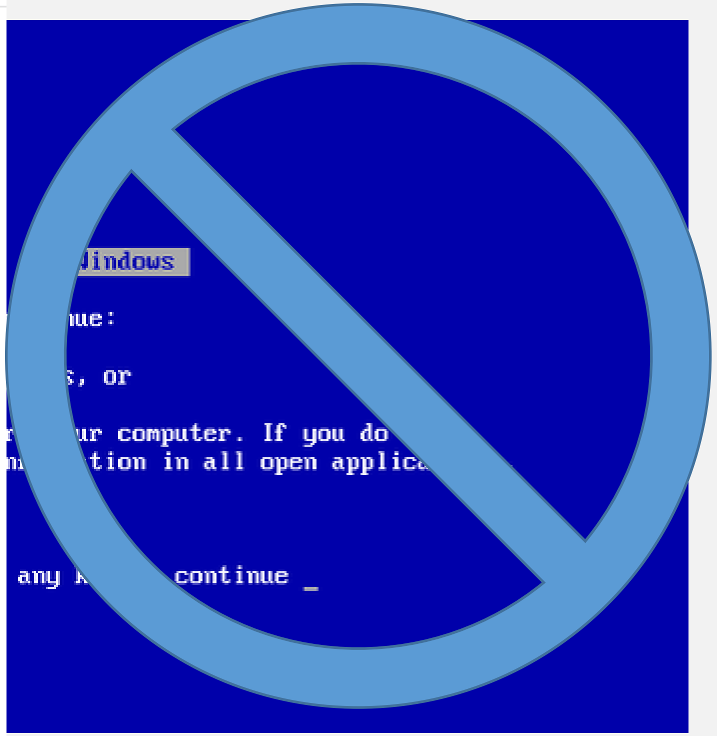
Static Driver Verifier Research Platform README

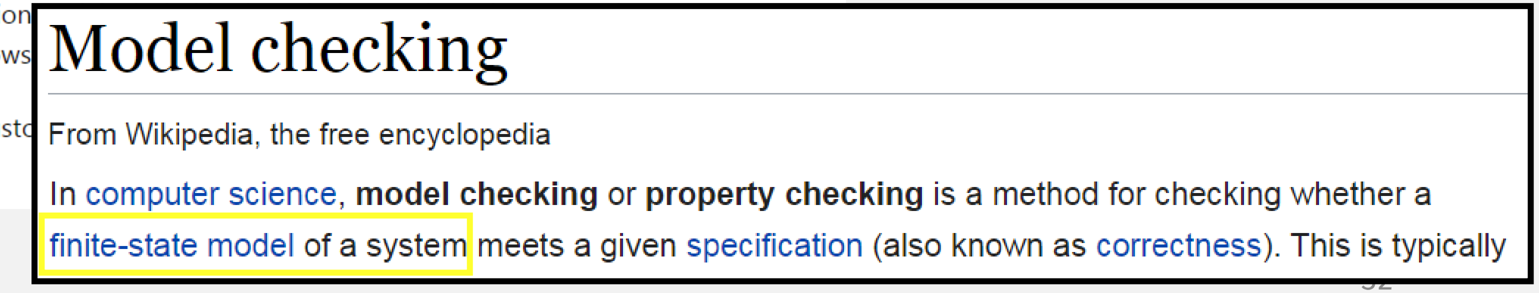
Overview of Static Driver Verifier Research Platform

Static Driver Verifier (SDV) is a compile-time static verification

Research Platform (SDVRP) is an extension to SDV that allows

- Support additional frameworks (or APIs) and write custo
- Experiment with the model checking step.





Model checking

From Wikipedia, the free encyclopedia

In computer science, **model checking** or **property checking** is a method for checking whether a finite-state model of a system meets a given specification (also known as correctness). This is typically

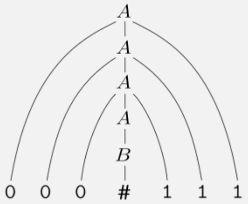
Algorithms (Decidable Problems) for Context-Free Languages (CFLs)

<u>Thm</u>: A_{CFG} is a decidable language

$$A_{\mathsf{CFG}} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}$$

- This a is very practically important problem …
- … equivalent to:
  - Is there an **algorithm to <u>parse</u> a programming language** with grammar G?

- A Decider for this problem could … ?
  - Try every possible derivation of G, and check if it's equal to w?
  - But this might never halt
    - E.g., what if there is a rule like: $\mathbf{S} \rightarrow \mathbf{0S}$ or $\mathbf{S} \rightarrow \mathbf{S}$
  - This TM would be a recognizer <u>but not a decider</u>

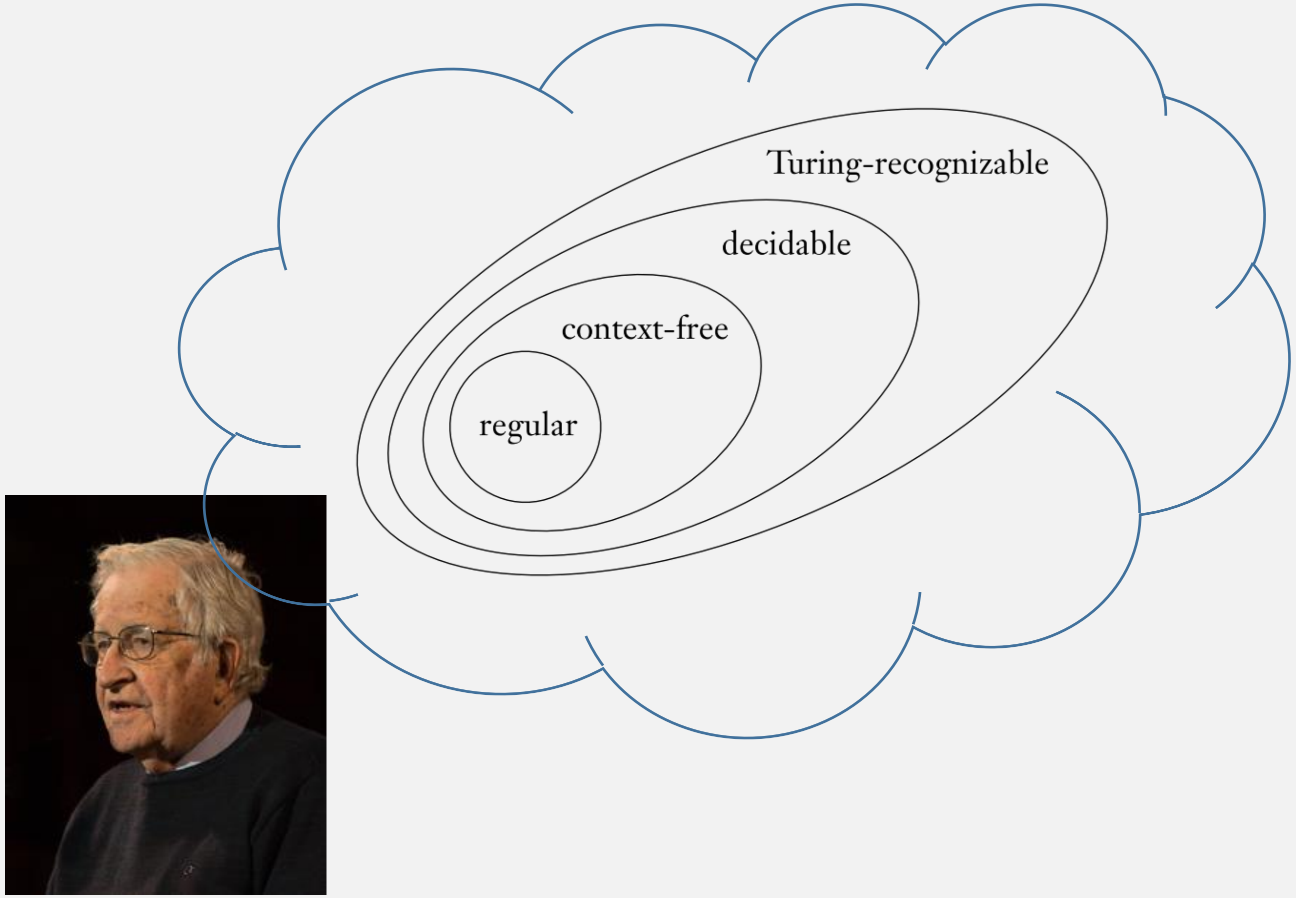


<u>Idea</u>: can the TM stop checking after some length?

- I.e., Is there upper bound on the number of derivation steps?

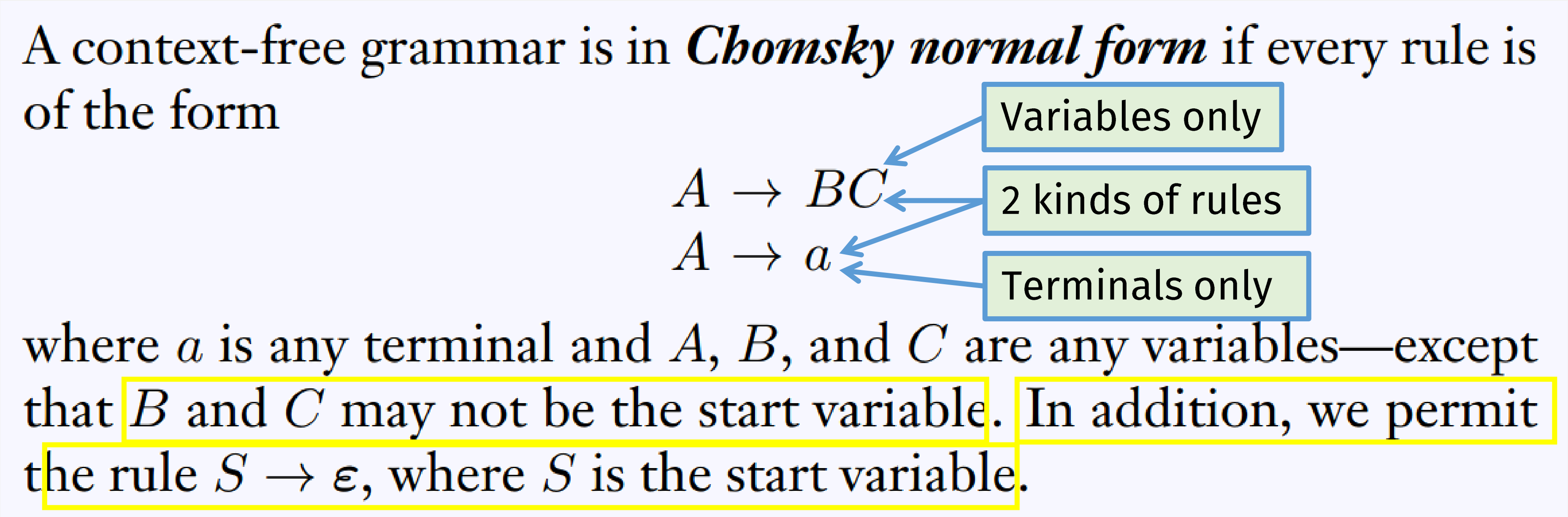
Chomsky Normal Form

Noam Chomsky



- He (sort of) invented this course too!

Chomsky Normal Form



A context-free grammar is in ***Chomsky normal form*** if every rule is of the form

$$A \to BC$$
$$A \to a$$

where a is any terminal and A, B, and C are any variables—except that B and C may not be the start variable. In addition, we permit the rule $S \to \varepsilon$, where S is the start variable.

Variables only

2 kinds of rules

Terminals only

Chomsky Normal Form: Number of Steps

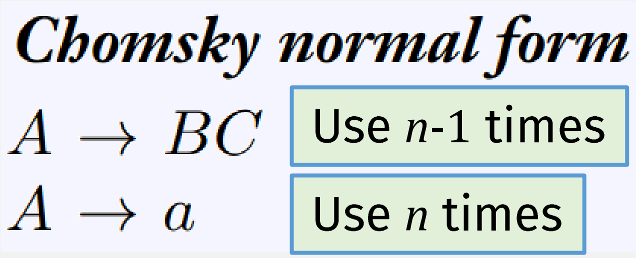
To generate a string of length n:

$n - 1$ steps: to generate n variables

+ n steps: to turn each variable into a terminal

Total: $2n - 1$ steps

(A **finite** number of steps)

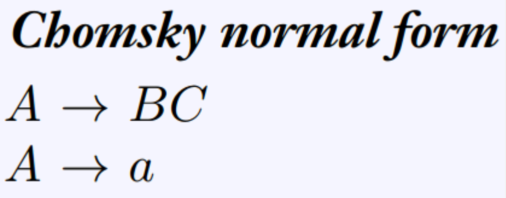


Chomsky normal form

$A \to BC$ Use n-1 times

$A \to a$ Use n times

Thm: Every CFG has a Chomsky Normal Form



Chomsky normal form

$A \to BC$

$A \to a$

1. Add new start variable S_0 that does not appear on any RHS
    - I.e., add rule $S_0 \to S$, where S is old start var

$$\begin{aligned} S &\to ASA \mid \mathtt{a}B \\ A &\to B \mid S \\ B &\to \mathtt{b} \mid \varepsilon \end{aligned}$$

➡

$$\begin{aligned} \boxed{\boldsymbol{S_0} \to \boldsymbol{S}} \\ S &\to ASA \mid \mathtt{a}B \\ A &\to B \mid S \\ B &\to \mathtt{b} \mid \varepsilon \end{aligned}$$

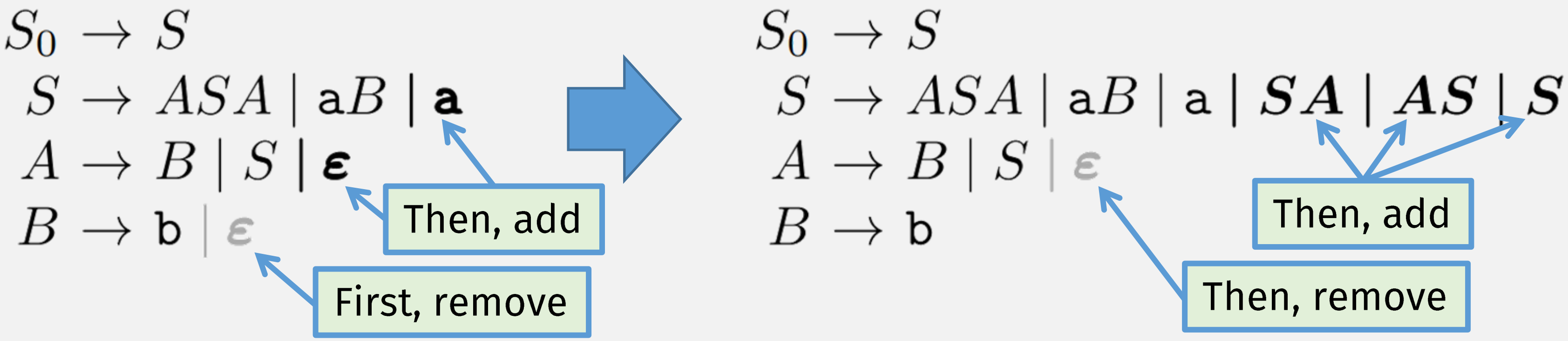
Thm: Every CFG has a Chomsky Normal Form

Chomsky normal form

$A \to BC$

$A \to a$

1. Add new start variable S_0 that does not appear on any RHS
   - I.e., add rule $S_0 \to S$, where S is old start var
2. Remove all "empty" rules of the form $A \to \varepsilon$
   - A must not be the start variable
   - Then for every rule with A on RHS, add new rule with A deleted
     - E.g., If $R \to uAv$ is a rule, add $R \to uv$
   - Must cover all combinations if A appears more than once in a RHS
     - E.g., if $R \to uAvAw$ is a rule, add 3 rules: $R \to uvAw$, $R \to uAvw$, $R \to uvw$



$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid \mathrm{a}B \mid \mathbf{a} \\
A &\to B \mid S \mid \boldsymbol{\varepsilon} \\
B &\to \mathrm{b} \mid \varepsilon
\end{aligned}
$$

Then, add

First, remove

$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid \mathrm{a}B \mid \mathrm{a} \mid \boldsymbol{SA} \mid \boldsymbol{AS} \mid \boldsymbol{S} \\
A &\to B \mid S \mid \varepsilon \\
B &\to \mathrm{b}
\end{aligned}
$$

Then, add

Then, remove

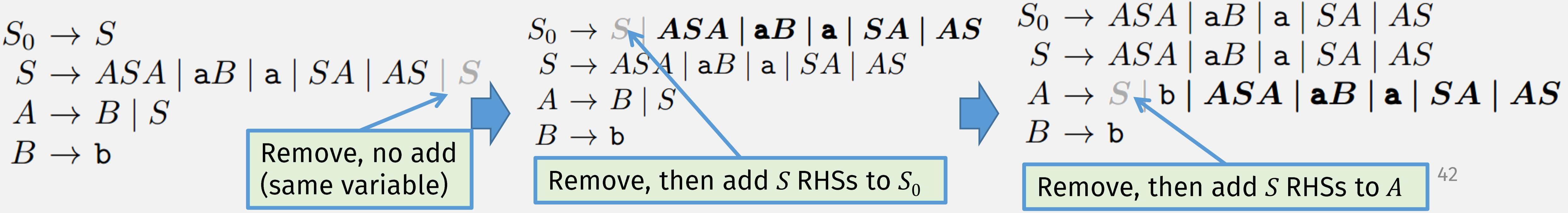
Thm: Every CFG has a Chomsky Normal Form

Chomsky normal form

$A \to BC$

$A \to a$

1. Add new start variable S_0 that does not appear on any RHS
   - I.e., add rule $S_0 \rightarrow S$, where S is old start var
2. Remove all "empty" rules of the form $A \rightarrow \varepsilon$
   - A must not be the start variable
   - Then for every rule with A on RHS, add new rule with A deleted
     - E.g., If $R \rightarrow uAv$ is a rule, add $R \rightarrow uv$
   - Must cover all combinations if A appears more than once in a RHS
     - E.g., if $R \rightarrow uAvAw$ is a rule, add 3 rules: $R \rightarrow uvAw$, $R \rightarrow uAvw$, R -> uvw
3. Remove all "unit" rules of the form $A \rightarrow B$
   - Then, for every rule $B \rightarrow u$, add rule $A \rightarrow u$



$$\begin{aligned} S_0 &\to S \\ S &\to ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \mid S \\ A &\to B \mid S \\ B &\to \mathtt{b} \end{aligned}$$

Remove, no add (same variable)

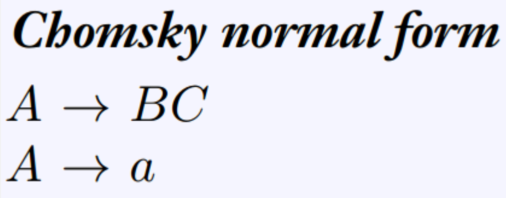
$$\begin{aligned} S_0 &\to S \mid \boldsymbol{ASA} \mid \mathbf{a}\boldsymbol{B} \mid \mathbf{a} \mid \boldsymbol{SA} \mid \boldsymbol{AS} \\ S &\to ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \\ A &\to B \mid S \\ B &\to \mathtt{b} \end{aligned}$$

Remove, then add S RHSs to S_0

$$\begin{aligned} S_0 &\to ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \\ S &\to ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \\ A &\to S \mid \mathtt{b} \mid \boldsymbol{ASA} \mid \mathbf{a}\boldsymbol{B} \mid \mathbf{a} \mid \boldsymbol{SA} \mid \boldsymbol{AS} \\ B &\to \mathtt{b} \end{aligned}$$

Remove, then add S RHSs to A

Thm: Every CFG has a Chomsky Normal Form

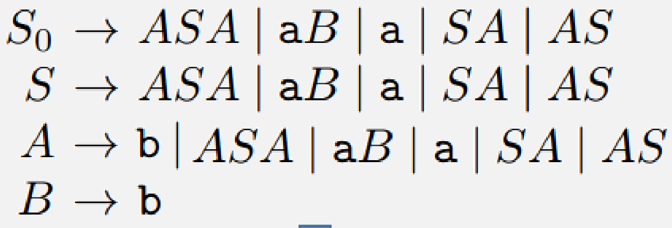


Chomsky normal form

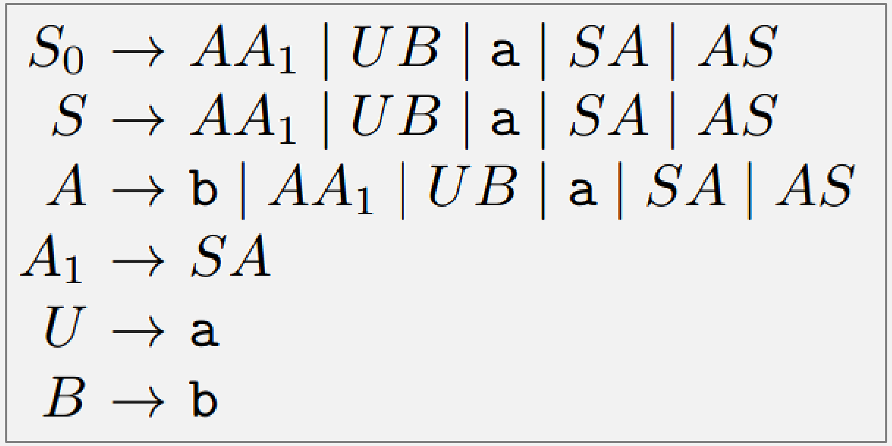
$A \to BC$

$A \to a$

1. Add new start variable S_0 that does not appear on any RHS
   - I.e., add rule $S_0 \to S$, where S is old start var
2. Remove all “empty” rules of the form $A \to \varepsilon$
   - A must not be the start variable
   - Then for every rule with A on RHS, add new rule with A deleted
     - E.g., If $R \to uAv$ is a rule, add $R \to uv$
   - Must cover all combinations if A appears more than once in a RHS
     - E.g., if $R \to uAvAw$ is a rule, add 3 rules: $R \to uvAw$, $R \to uAvw$, R -> uvw
3. Remove all “unit” rules of the form $A \to B$
   - Then, for every rule $B \to u$, add rule $A \to u$
4. Split up rules with RHS longer than length 2
   - E.g., $A \to wxyz$ becomes $A \to wB$, $B \to xC$, $C \to yz$
5. Replace all terminals on RHS with new rule
   - E.g., for above, add $W \to w$, $X \to x$, $Y \to y$, $Z \to z$



$$\begin{aligned} S_0 &\to ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \\ S &\to ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \\ A &\to \mathtt{b} \mid ASA \mid \mathtt{a}B \mid \mathtt{a} \mid SA \mid AS \\ B &\to \mathtt{b} \end{aligned}$$



$$\begin{aligned} S_0 &\to AA_1 \mid UB \mid \mathtt{a} \mid SA \mid AS \\ S &\to AA_1 \mid UB \mid \mathtt{a} \mid SA \mid AS \\ A &\to \mathtt{b} \mid AA_1 \mid UB \mid \mathtt{a} \mid SA \mid AS \\ A_1 &\to SA \\ U &\to \mathtt{a} \\ B &\to \mathtt{b} \end{aligned}$$

Thm: A_{CFG} is a decidable language

$$A_{\text{CFG}} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}$$

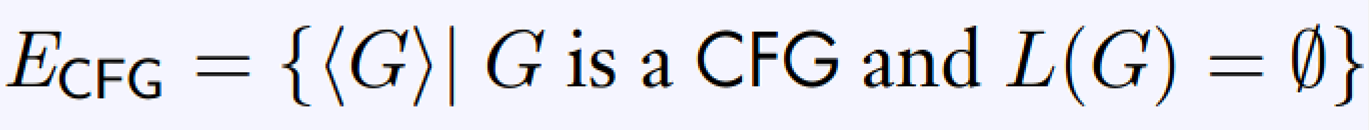
Proof: create the decider:

$S =$ "On input $\langle G, w\rangle$, where G is a CFG and w is a string:

1. Convert G to an equivalent grammar in Chomsky normal form.
2. List all derivations with $2n - 1$ steps, where n is the length of w; except if $n = 0$, then instead list all derivations with one step.
3. If any of these derivations generate w, *accept*; if not, *reject*."

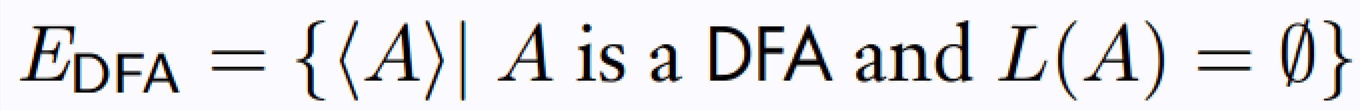
Termination argument?

Thm: E_{CFG} is a decidable language

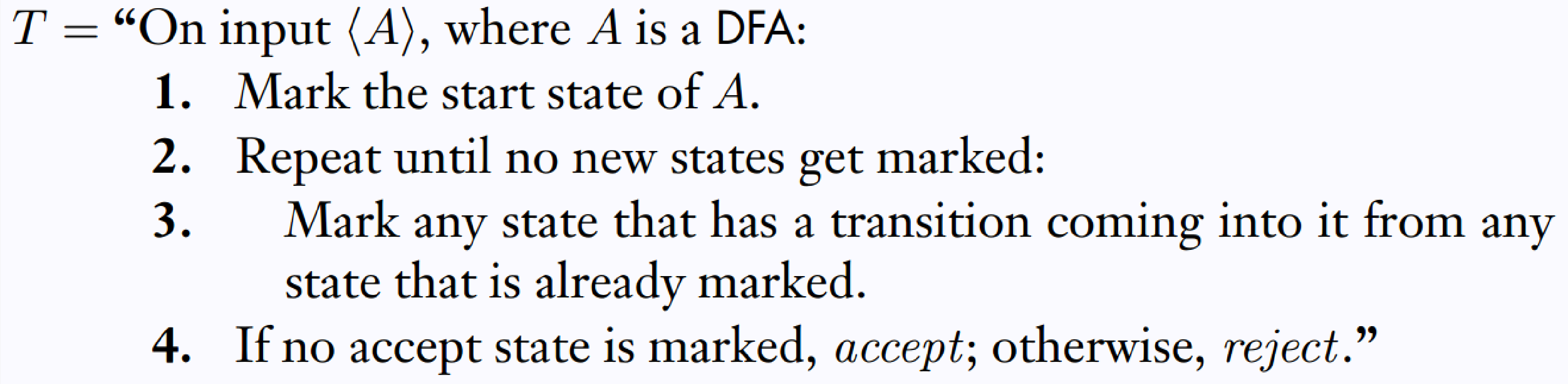


$$E_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$$

Recall:



$$E_{\mathsf{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$$



```
T = "On input ⟨A⟩, where A is a DFA:
    1. Mark the start state of A.
    2. Repeat until no new states get marked:
    3.     Mark any state that has a transition coming into it from any
           state that is already marked.
    4. If no accept state is marked, accept; otherwise, reject."
```

"Reachability" (of accept state from start state) algorithm

Thm: E_{CFG} is a decidable language

$E_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$

- Create decider that calculates reachability for grammar G
  - Except go backwards, start from terminals, to avoid looping

```
R = "On input ⟨G⟩, where G is a CFG:
    1. Mark all terminal symbols in G.
    2. Repeat until no new variables get marked:
    3.     Mark any variable A where G has a rule A → U_1U_2 ··· U_k and
           each symbol U_1, ..., U_k has already been marked.
    4. If the start variable is not marked, accept; otherwise, reject."
```

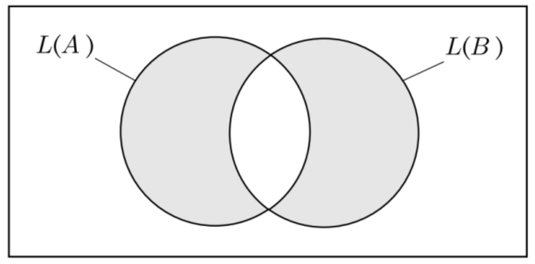
Termination argument?

Thm: EQ_{CFG} is a decidable language?

$EQ_{\text{CFG}} = \{\langle G, H\rangle |\ G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$

Recall: $EQ_{\text{DFA}} = \{\langle A, B\rangle |\ A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$

- Used Symmetric Difference



$$L(C) = \emptyset \text{ iff } L(A) = L(B)$$

  - where C = complement, union, intersection of machines A and B
- Can't do this for CFLs!
  - Intersection and complement are not closed for CFLs!!!

Intersection of CFLs is Not Closed!

- If closed, then intersection of these CFLs should be a CFL:

$$A = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n \mid m, n \geq 0\}$$

$$B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^m \mid m, n \geq 0\}$$

- But $A \cap B$ = $\{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\}$

- Not a CFL!

Complement of a CFL is not Closed!

- If CFLs closed under complement:

$$\text{if } G_1 \text{ and } G_2 \text{ context-free}$$

$$\overline{L(G_1)} \text{ and } \overline{L(G_2)} \text{ context-free}$$

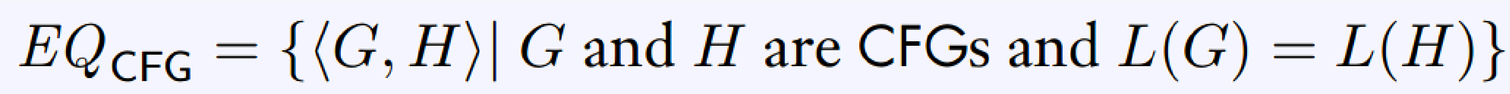
$$\overline{L(G_1)} \cup \overline{L(G_1)} \text{ context-free}$$

$$\overline{\overline{L(G_1)} \cup \overline{L(G_1)}} \text{ context-free}$$

$$L(G_1) \cap L(G_2) \text{ context-free}$$

DeMorgan's Law!

<u>Thm</u>: EQ_{CFG} is a decidable language?



$$EQ_{\mathsf{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are } \mathsf{CFGs} \text{ and } L(G) = L(H)\}$$

- No!
  - You cannot decide whether two grammars represent the same lang!

- It’s not recognizable either!
  - (We don’t know how to prove this yet)

Decidability of CFGs Recap

- $A_{\mathsf{CFG}} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}$
  - Convert grammar to Chomsky Normal Form
  - Then check all possible derivations of length 2|*w*| - 1 steps
- $E_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$
  - Compute “reachability” of start variable from terminals
- $EQ_{\mathsf{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$
  - We couldn’t prove that this is decidable!
  - (So you cant use this theorem when creating another decider)

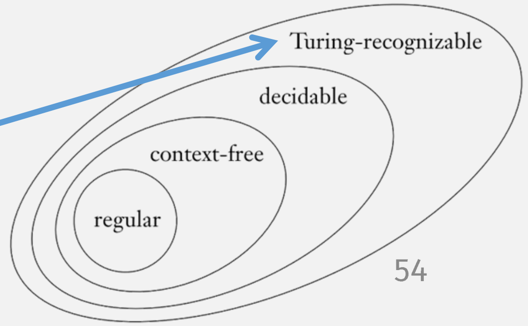
The Limits of Turing Machines?

- So TMs can express any “computation”
  - I.e., any (Python, Java, …) program you write is a Turing Machine

- So why do we focus on TMs that process other machines?

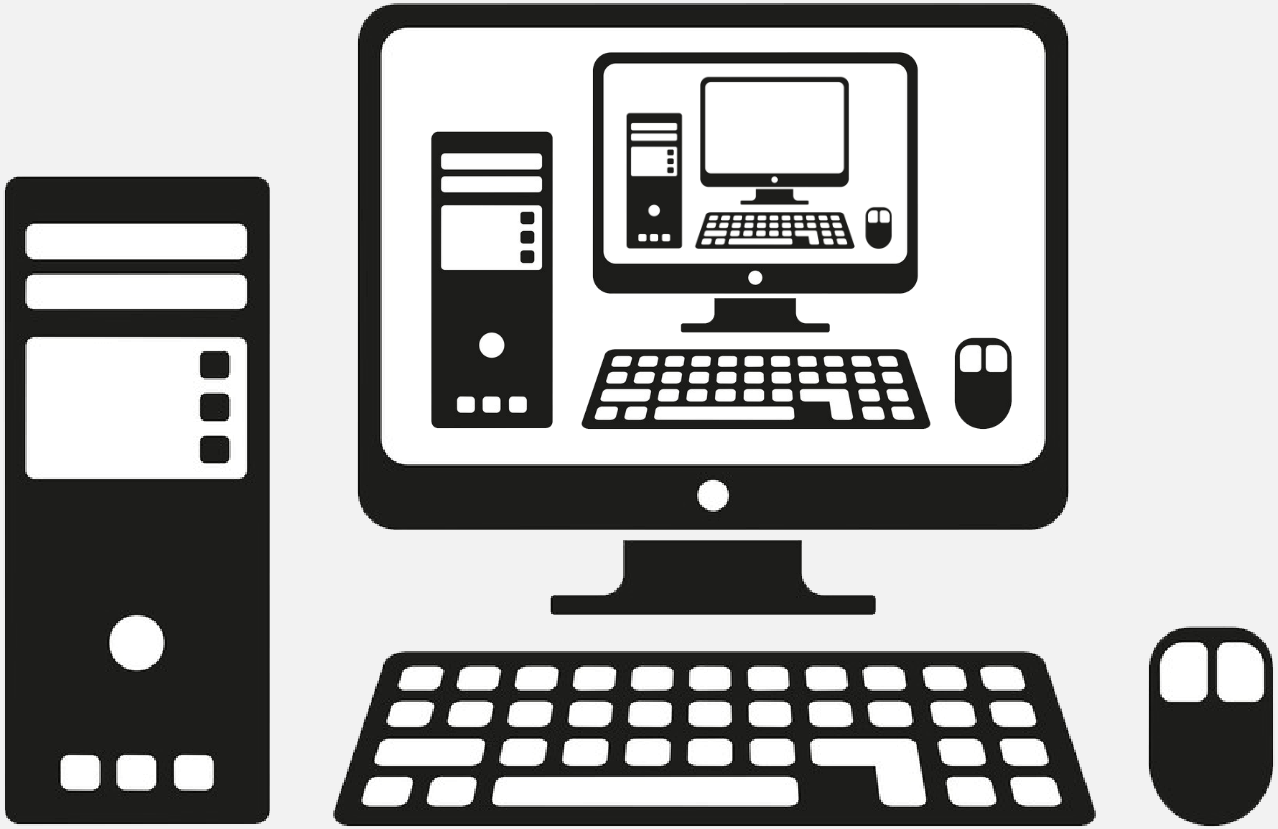
- Because we also want to study the <u>limits</u> of computation
  - And a good way to test the limit of a computational model is to see what it can compute about other computational models …

- So what are the limits of TMs? I.e., what’s here?
  - Or out here?



Next time: A_{TM} is undecidable ???

$A_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$



Check-in Quiz 10/18

On gradescope