

Welcome to CS622!

Theory of Formal Languages

UMass Boston Computer Science

Instructor: Stephen Chang

Friday, January 26, 2024

Lecture 3

Last Time

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Analogy:

Programming Language



Computation Model

(system of definitions and rules)

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“Theory” + “Formal” = math
(This is a math course!)

Analogy:

Programming Language



Computation Model

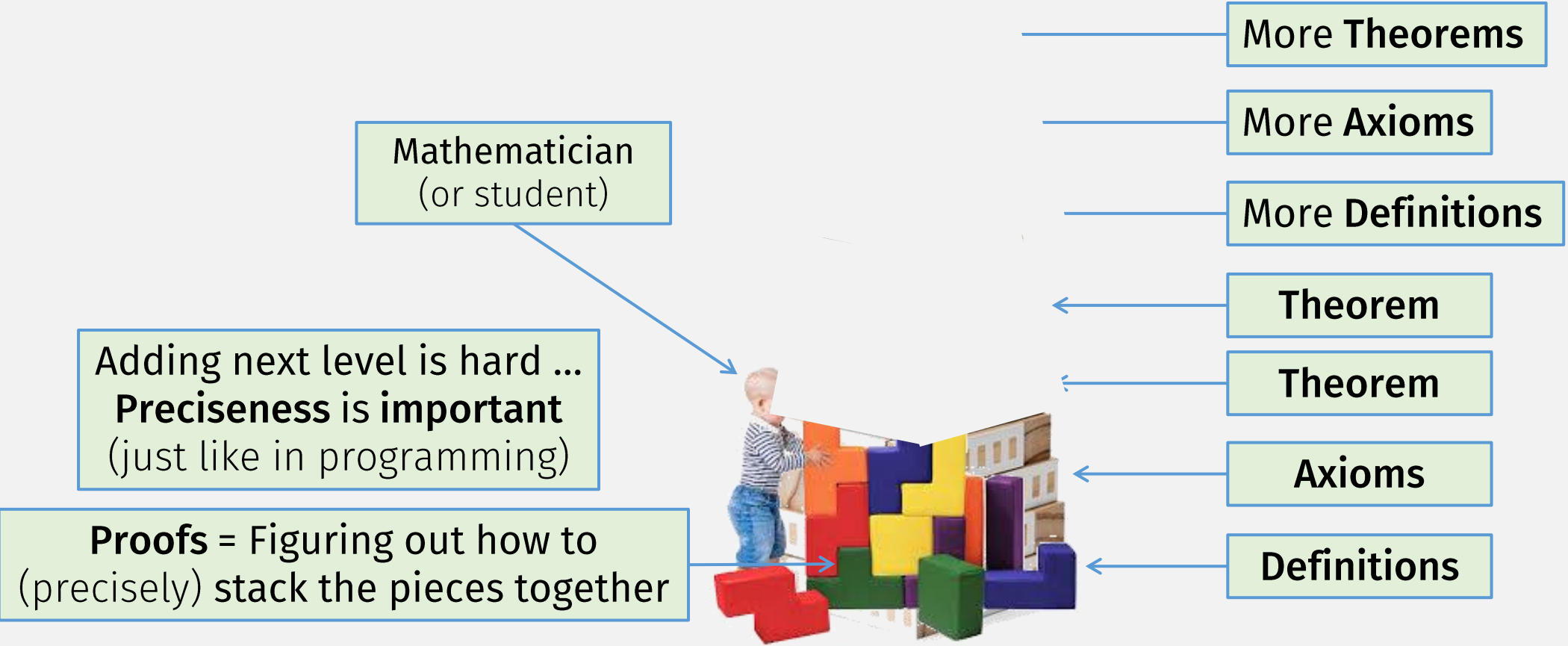
(system of definitions and rules)

A precisely defined

In CS 622 this semester, we will ...

1. Formally define and **study models of computation**
 - models will be as *simple* as possible (to make them easier to study)
2. Compare & contrast models of computation
 - which “programs” are *included / excluded* by a model
 - *Equality* or *overlap* between models?
3. Prove things about the models

How Mathematics (Proofs) Work



The “Modus Ponens” Inference Rule

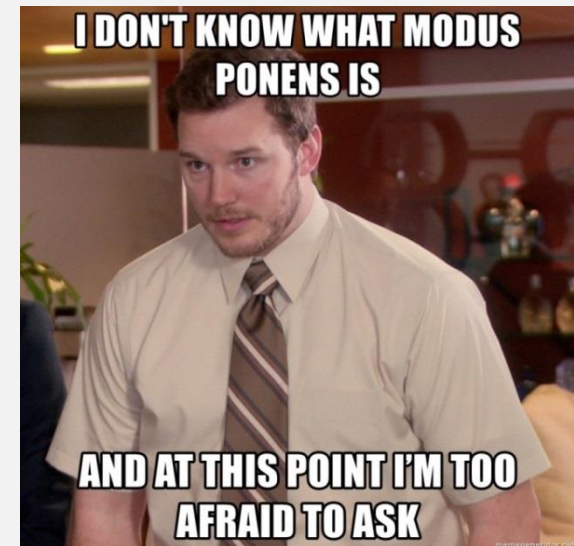
(Precisely Fitting Blocks Together)

Premises (if we can show these statements are true)

- If P then Q
- P is TRUE

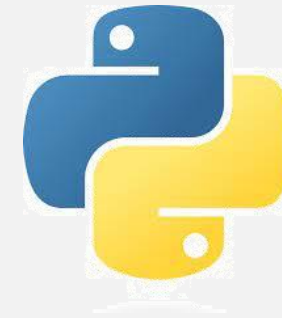
Conclusion (then we can say that this is also true)

- Q must also be TRUE



You already do “Proof” when Programming

```
def f(x):  
    if (x > 0) | (x < 0) | (x == 0):  
        return x + 1  
    else:  
        return 1 / 0
```



Can this function ever throw ZeroDivisionError?

How did you figure out the answer?

You did a proof!

(Let's write it out formally)

Deductive Proof Example

Prove: fn f never throws ZeroDivisionError

```
def f(x):  
    "test expr"  
    if (x > 0) | (x < 0) | (x == 0):  
        return x + 1 "first branch"  
    else:  
        return 1 / 0 "second branch"
```

Proof:

Prior steps are already-proved, can be used to prove later steps!

Statements / Justifications Table

Statements

1. If running "test expr" is True, then "first branch" runs
2. If running "test expr" is False, then "second branch" runs
3. running "test expr" is (always) True
- 4. "first branch" (always) runs

Justifications

1. Rules of Python
2. Rules of Python
3. Definition of "numbers"
4. By steps 1, 3, and modus ponens

Modus Ponens
If we can prove these:

- If P then Q
- P

Then we've proved:

- Q ←

7. fn f never throws ZeroDivisionError

Deductive Proof Example

Prove: fn f never throws ZeroDivisionError

```
def f(x):  
    if (x > 0) | (x < 0) | (x == 0):  
        return x + 1 "first branch"  
    else:  
        return 1 / 0 "second branch"
```

Proof:

Statements / Justifications Table

Statements

1. If running "test expr" is True, then "first branch" runs
2. If running "test expr" is False, then "second branch" runs
3. running "test expr" is (always) True
4. "first branch" (always) runs
5. "second branch" never runs
6. fn f never runs 1 / 0
- ➔ 7. fn f never throws ZeroDivisionError

Justifications

1. Rules of Python
2. Rules of Python
3. Definition of "numbers"
4. By steps 1, 3, and modus ponens
5. By step 4, and Rules of Python?
6. By step 5
7. By step 6 and ???

What else can we prove about programs?

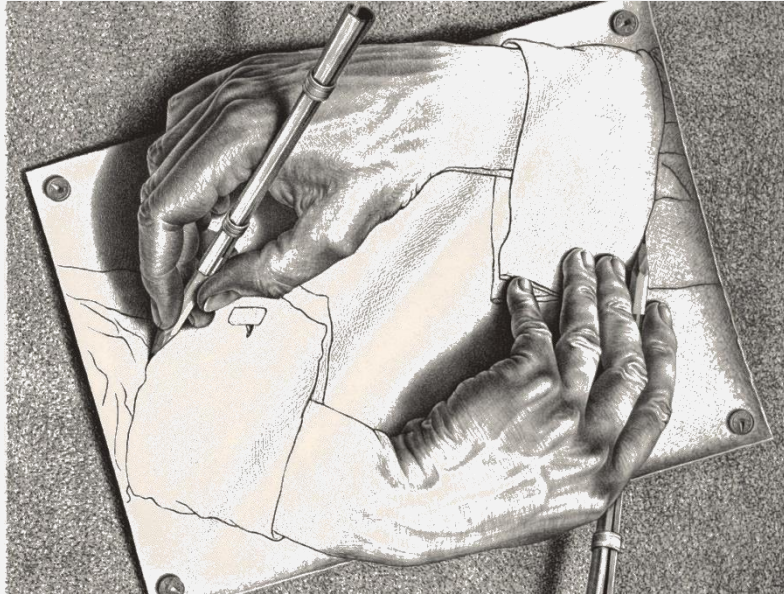
```
function check(n)
{ // check if the number n is a prime
  var factor; // if the checked number is not a prime, this is its first factor
  var c;
  factor = 0;
  // try to divide the checked number by all numbers till its square root
  for (c=2; (c <= Math.sqrt(n)); c++)
  {
    if (n%c == 0) // is n divisible by c ?
      { factor = c; break }
  }
  return (factor);
} // end of check function

function communicate()
{ // communicate with the user
  var i; // i is the checked number
  var factor; // if the checked number is not a prime, this is its first factor
  i = document.primetest.number.value; // get the checked number
  // is it a valid input?
  if ((i != N(i)) || (i <= 0) || (Math.floor(i) != i))
    { alert ("The checked object should be a whole positive number"); }
  else
  {
    factor = check (i);
    if (factor == 0)
      { alert (i + " is a prime number"); }
    else
      { alert (i + " is not a prime number. i =" + factor + "X" + i/factor) }
  }
} // end of communicate function
```



Predict result without running a program?

Can we make predictions about computation?



It's tricky: **Trying to predict computation requires computation!**

Can we make predictions about computation?

- The **Halting Lemma** says:



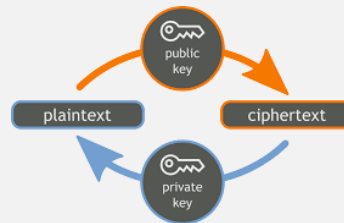
- And **Rice's Theorem** says:

- “all non-trivial, semantic properties of programs are undecidable”

Knowing What Computers Can't Do is Still Useful!

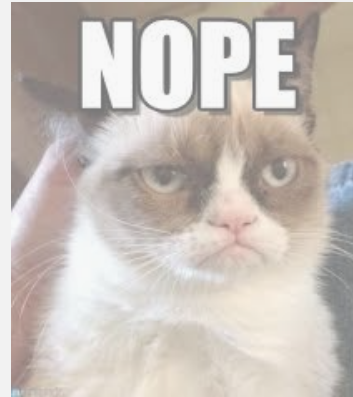
In Cryptography:

- Perfect secrecy is impossible in practice
- But with slightly imperfect secrecy (i.e., a computationally bounded adversary) we get:



Can we make predictions about computation?

- The **Halting Lemma** says:



- And **Rice's Theorem** says:

- “all non-trivial, semantic properties of programs are undecidable”

Actually:

- it depends on the computation model!



Predicting What Some Programs Will Do ...

microsoft.com/en-us/research/project/slam/

SLAM is a project for checking that software satisfies critical behavioral properties of the interfaces it uses and to aid software engineers in designing interfaces and software that ensure reliable and correct functioning. Static Driver Verifier is a tool in the Windows Driver Development Kit that uses the SLAM verification engine.

"Things like even software verification, this has been the Holy Grail of computer science for many decades but now in some very key areas, for example, driver verification we're building tools that can do actual proof about the software and how it works in order to guarantee the reliability." **Bill Gates**, April 18, 2002. [Keynote address at WinHec 2002](#)

SLAM
Microsoft Research
{ocs, end()}node}

Predicting things about programs ... is the Holy grail of CS!

Static Driver Verifier Research Platform README

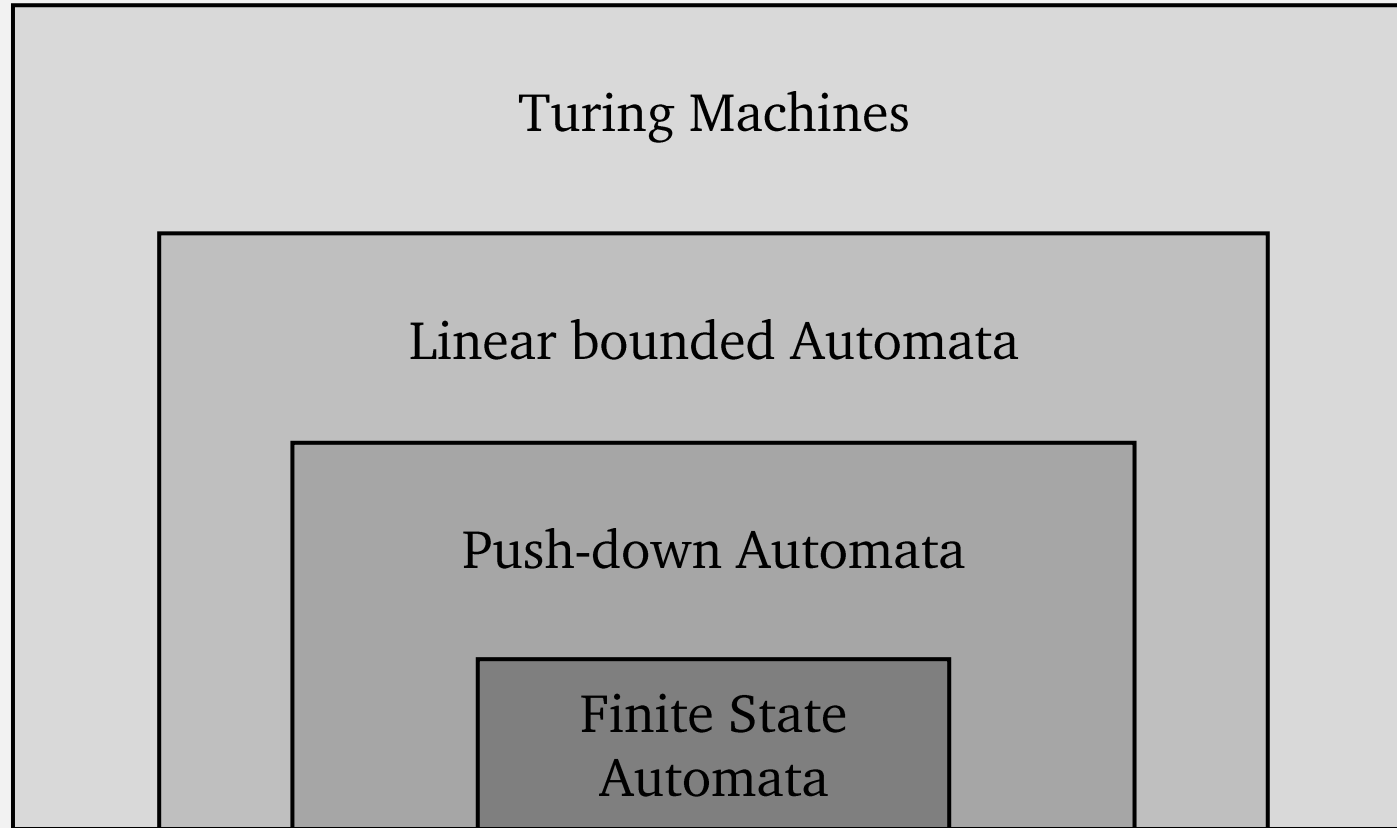
Overview of Static Driver Verifier Research Platform

Static Driver Verifier (SDV) is a compile-time static verification tool, included in the Windows Driver Kit (WDK). The SDV Research Platform (SDVRP) is an extension to SDV that allows you to adapt SDV to:

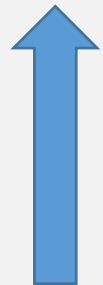
- Support additional frameworks (or APIs) and write custom SLIC rules for this framework.
- Experiment with the model checking step.



CS 420 Proofs About Computational Models

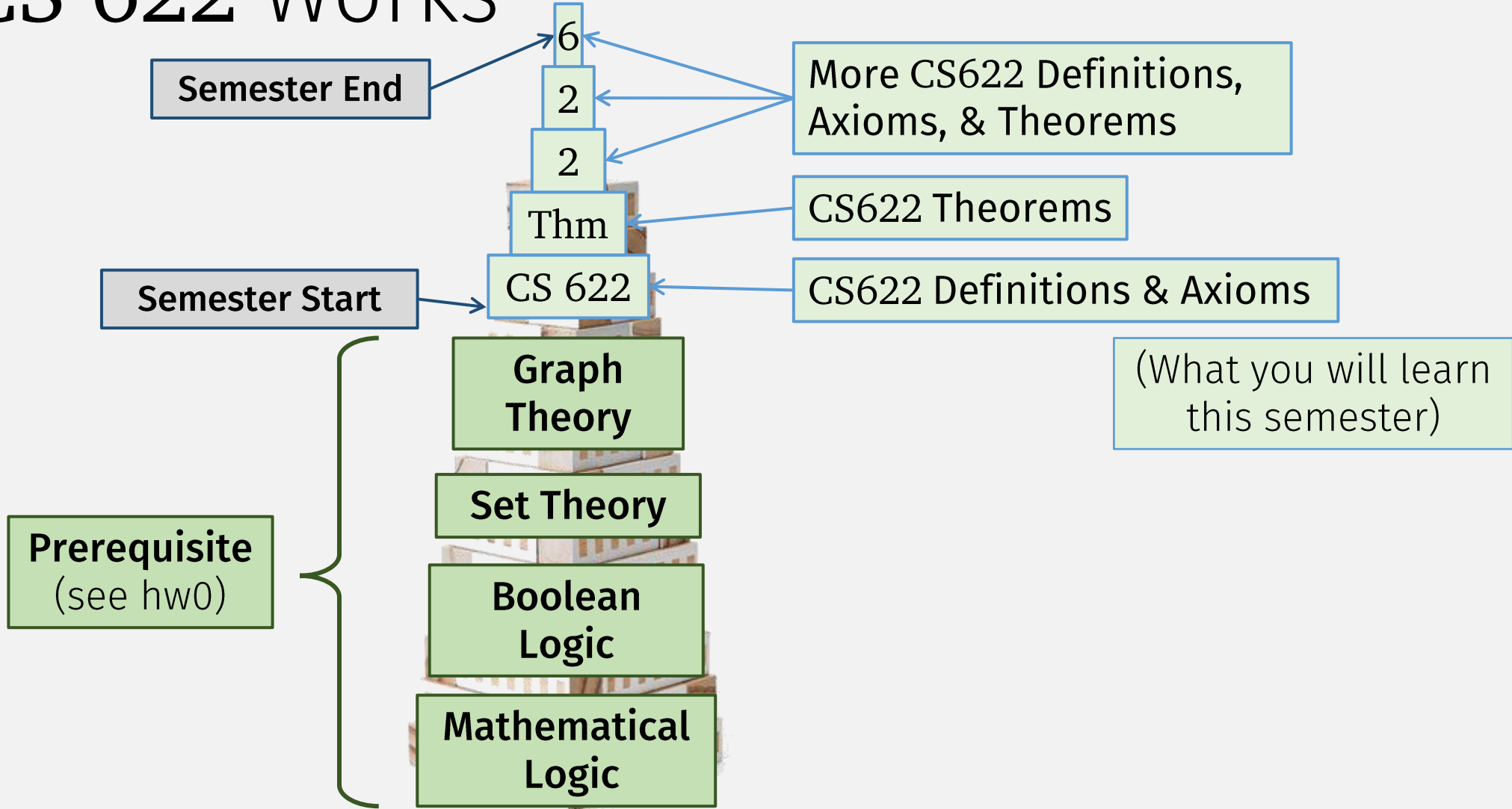


More powerful
More complex
Less restricted



In this class, we will prove things about our simple computational models

How CS 622 Works



A Word of Advice

Important:
Do not fall behind
in this course



To prove a (new) theorem ...

... need to know all axioms,
definitions, and (previous)
theorems below it

Another Word of Advice

HW 1, Problem 1

Prove that $ABC = XYZ$



How can I help you today?

Message ChatGPT... Prove that $ABC = XYZ$

A Not-From-CS621 -
Spring2024 Theorem



“Blocks” from outside the course won’t work in the proof

Remember:

Preciseness in proofs (just like in programming) **is critical**
(Proofs must connect facts from this course exactly)



... can be used to **prove** (new) **theorems** in this course

Only axioms, definitions, and **theorems** from this course...

HW problems are *graded* on precise steps in the proof, not on the final theorem itself!

Textbooks

- Sipser. *Intro to Theory of Computation*, 3rd ed.
- Hopcroft, Motwani, Ullman. *Intro to Automata Theory, Languages, and Computation*, 3rd ed.

Strongly Recommended (but not required)

- Slides (posted) and lecture should be **self-contained**
- BUT, **Students who do well read the book**

All course info available on web site:
<https://www.cs.umb.edu/~stchang/cs622/s24>

How to Do Well in this Course

- Learn the “ building blocks”
 - I.e., axioms, definitions, and theorems
- To solve a problem (prove a new theorem) ...
... think about how to (precisely) combine existing “blocks”
- HW problems graded on steps to the answer (not final theorem)
- Don't Fall Behind!
 - Start HW Early (HW 0 due Monday 1/22 12pm EST noon)
- Participate and Engage
 - Lecture
 - Office Hours
 - Message Boards (piazza)

Grading

- **HW: 80%**
 - Weekly: In / Out Monday
 - Approx. 12 assignments
 - Lowest grade dropped
- **Participation: 20%**
 - Lecture participation, in-class work, office hours, piazza
- **No exams**
- **A range: 90-100**
- **B range: 80-90**
- **C range: 70-80**
- **D range: 60-70**
- **F: < 60**

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Late HW

- Is bad ... try not to do it please
 - Grades get delayed
 - Can't discuss solutions
 - You fall behind!
- Late Policy: **3 late days** to use during the semester

HW Collaboration Policy

Allowed

- Discussing HW with classmates (but must cite)
- Using other resources to learn, e.g., youtube, other textbooks, ...
- Writing up answers on your own, from scratch, in your own words

Not Allowed

- Submitting someone else's answer
- Submitting someone else's answer with:
 - variables changed,
 - thesaurus words,
 - or sentences rearranged ...
- Using sites like Chegg, CourseHero, Bartleby, Study, ChatGPT, etc.
- Using theorems or definitions not from this course

Honesty Policy

- 1st offense: zero on problem
- 2nd offense: zero on hw, reported to school
- 3rd offense+: F for course

Regret policy

- If you self-report an honesty violation, you'll only receive a zero on the problem and we move on.

All Up to Date Course Info

Survey, Schedule, Office Hours, HWs, ...

See course website:

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hw0 (pre-req quiz)
(see gradescope)