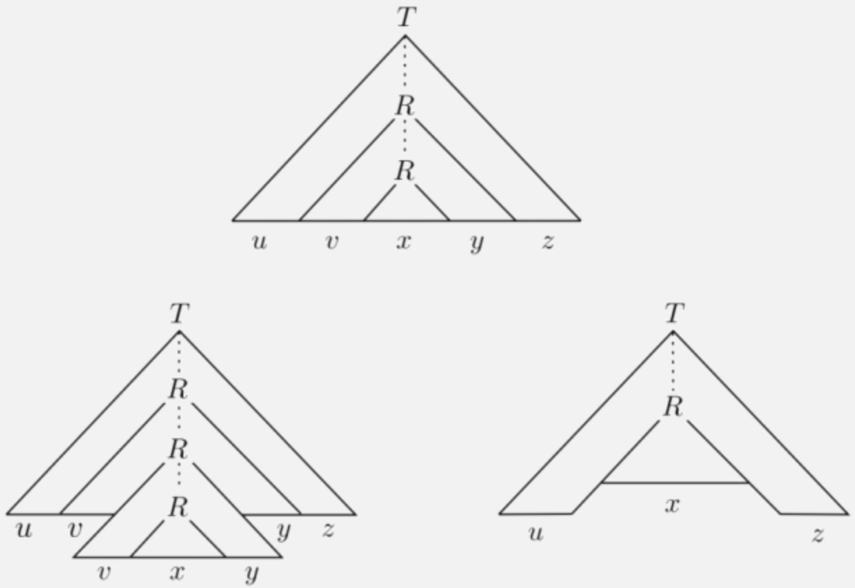
UMB CS 622

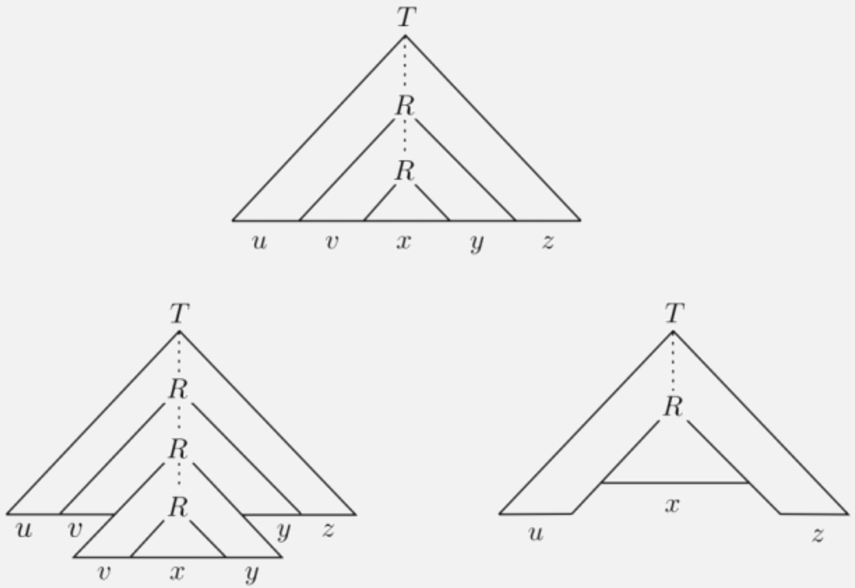
CFL Pumping Lemma

Friday, March 29, 2024



Announcements

- HW 6
  - Due Monday 4/1 12pm noon



Last Time

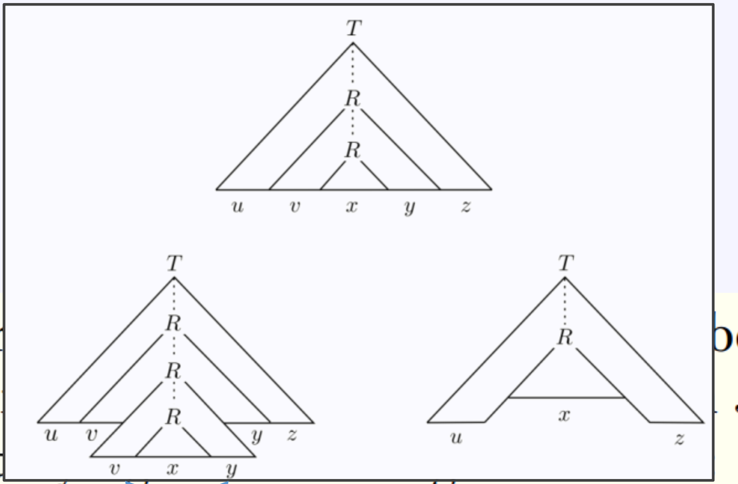
Pumping Lemma for CFLS

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

Two pumpable parts.
But they must be pumped together!

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Pumping lemma If A is a regular ... ber p (the pumping length) where if s is any stri... s may be divided into three pieces, $s = xyz$, sat...



1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

One pumpable part

Two pumpable parts, pumped together

Previously

A Non CFL example

language $B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n | \, n \geq 0\}$ is not context free

Intuition

- **Strings in CFLs** can have **two parts** that are **"pumped" together**
- **Language** B **requires** **three parts** to be **"pumped" together**
- So **it's not a CFL!**

Proof?

Want to prove: $a^n b^n c^n$ **is not** a CFL

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Reminder: CFL Pumping lemma says: all strings $a^n b^n c^n$ ≥ length p are splittable into $uvxyz$ where v and y are pumpable

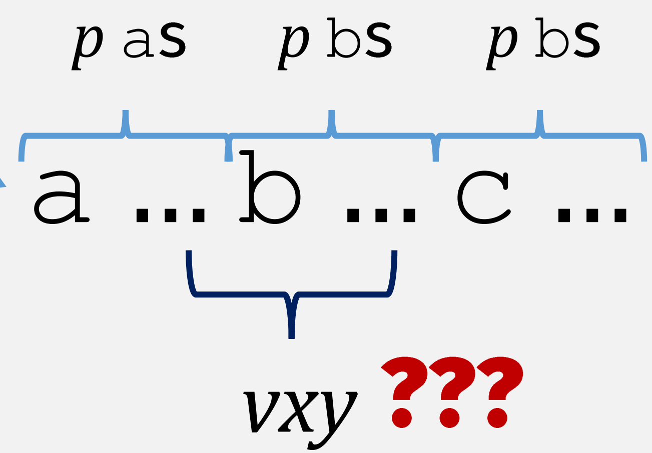
Proof (by contradiction):

Now we must find a contradiction ...

- Assume: $a^n b^n c^n$ **is** a CFL
  - So it must satisfy the pumping lemma for CFLs
  - I.e., all strings ≥ length p are pumpable
- Counterexample = $a^p b^p c^p$

Contradiction if:
- A string in the language ☑
- ≥ length p ☑
- Is **not splittable** into $uvxyz$ where v and y are pumpable

???



Want to prove: $a^n b^n c^n$ **is not** a CFL

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Possible Splits

Proof (by contradiction):

- **Assume:** $a^n b^n c^n$ **is** a CFL
  - So it must satisfy the pumping lemma for CFLs
  - I.e., all strings $\geq$ length p are pumpable
- **Counterexample** = $a^p b^p c^p$

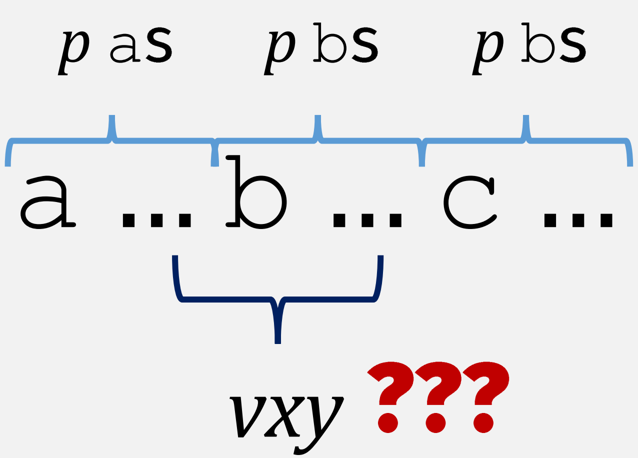
Contradiction if:
- A string in the language
- $\geq$ length p
- Is **not splittable** into $uvxyz$ where v and y are pumpable

contradiction

- Possible Splits (using condition # 3: $|vxy| \leq p$)

Not pumpable

- ☒ vxy is all as
- ☒ vxy is all bs
- ☒ vxy is all cs
- ☒ vxy has as and bs
- ☒ vxy has bs and cs
- (vxy cannot have as, bs, and cs --- bc of condition #3)



So $a^n b^n c^n$ is not a CFL
(justification:
contrapositive of CFL pumping lemma)

$a^p b^p c^p$ **cannot be split** into $uvxyz$
where v and y are pumpable!

Another Non-CFL $D = \{ww \mid w \in \{0,1\}^*\}$

Be careful when choosing **counterexample** *s*: 0^p10^p1

This *s* **can be pumped** according to **CFL pumping lemma:**

$$\overbrace{\underbrace{000\cdots000}_{u}\ \underbrace{0}_{v}\ \underbrace{1}_{x}}^{0^p1}\ \overbrace{\underbrace{0}_{y}\ \underbrace{000\cdots0001}_{z}}^{0^p1}$$

Pumping *v* and *y* (together) **produces string still in** *D* **...**

... just like **pumping lemma says** (no contradiction)**!**

- **CFL Pumping Lemma conditions:**
  - ☑ **1.** for each $i \geq 0$, $uv^ixy^iz \in A$,
  - ☑ **2.** $|vy| > 0$, and
  - ☑ **3.** $|vxy| \leq p$.

So **this attempt** to **prove** that the **language** is **not a CFL failed.**
(It doesn't prove that the language is a CFL!)

Another Non-CFL $D = \{ww \mid w \in \{0,1\}^*\}$

- Need <u>another counterexample</u> string s:

If vyx is contained in first or second half, then any pumping will break the match

$0^p 1^p 0^p 1^p$

So vyx must straddle the middle

But any pumping still breaks the match because order is wrong

e.g., $0^p 1^{p\text{-}1}$ 10 $0^{p\text{-}1} 1^p$

$0^p 1^{p\text{-}1}$ 10 10 $0^{p\text{-}1} 1^p$

- CFL Pumping Lemma conditions:

1. for each $i \geq 0$, $uv^ixy^iz \in A$,

2. $|vy| > 0$, and

3. $|vxy| \leq p$.

Now we have proven that this language is **not a CFL!**

A Practical Non-CFL

- **XML**
  - ELEMENT → <TAG>CONTENT</TAG>
  - Where TAG is any string

- XML also looks like this non-CFL: $D = \{ww \mid w \in \{0,1\}^*\}$

- This means XML is not context-free!
  - Note: HTML *is* context-free because …
  - … there are only a finite number of tags,
  - so they can be embedded into a finite number of rules.

In practice:

- XML is parsed as a CFL, with a CFG
- Then matching tags checked in a 2nd pass with a more powerful machine …

Next: A More Powerful Machine …

M_1 accepts its input if it is in language: $B = \{w\#w \mid w \in \{0,1\}^*\}$

M_1 = "On input string w:

1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether these positions contain the same symbol. If they do not, or if no # is found, *reject*. Cross off symbols as they are checked to keep track of which symbols correspond.

Infinite memory (initial contents are the **input string**)

Can move to, and **read/write from** <u>**arbitrary**</u> **memory locations!**